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Weighing Risk and Uncertainty

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Decision theory distinguishes between risky prospects, where the probabilities associated with the possible outcomes are assumed to be known, and uncertain prospects, where these probabilities are not assumed to be known. Studies of choice between risky prospects have suggested a nonlinear transformation of the probability scale that overweights low probabilities and underweights moderate and high probabilities. The present article extends this notion from risk to uncertainty by invoking the principle of bounded subadditivity: An event has greater impact when it turns impossibility into possibility, or possibility into certainty, than when it merely makes a possibility more or less likely. A series of studies provides support for this principle in decision under both risk and uncertainty and shows that people are less sensitive to uncertainty than to risk. Finally, the article discusses the relationship between probability judgments and decision weights and distinguishes relative sensitivity from ambiguity aversion.

Decisions are generally made without definite knowledge of their consequences. The decisions to invest in the stock market, to undergo a medical operation, or to go to court are generally made without knowing in advance whether the market will go up, the operation will be successful, or the court will decide in one's favor. Decision under uncertainty, therefore, calls for an evaluation of two attributes: the desirability of possible outcomes and their likelihood of occurrence. Indeed, much of the study of decision making is concerned with the assessment of these values and the manner in which they are—or should be—combined.

In the classical theory of decision under risk, the utility of each outcome is weighted by its probability of occurrence. Consider a simple prospect of the form (x, p) that offers a probability p to win $\$x$ and a probability $1 - p$ to win nothing. The expected utility of this prospect is given by $pu(x) + (1 - p)u(0)$, where u is the utility function for money. Expected utility theory has been developed to explain attitudes toward risk, namely, risk aversion and risk seeking. Risk aversion is defined as a preference for a sure outcome over a prospect with an equal or greater expected value. Thus, choosing a sure $100 over an even chance to win $200 or nothing is an expression of risk aversion. Risk seeking is exhibited if a prospect is preferred to a sure outcome with equal or greater expected value. It is commonly assumed that people are risk averse, which is explained in expected utility theory by a concave utility function.

The experimental study of decision under risk has shown that people often violate both the expected utility model and the principle of risk aversion that underlie much economic analysis. Table 1 illustrates a common pattern of risk seeking and risk aversion observed in choice between simple prospects (adapted from Tversky & Kahneman, 1992), where $C(x, p)$ is the median certainty equivalent of the prospect (x, p). Thus, the upper left-hand entry in the table shows that the median participant is indifferent between receiving $14 for sure and a 5% chance of receiving $100. Because the expected value of this prospect is only $5, this observation reflects risk seeking.

Table 1 illustrates a fourfold pattern of risk attitudes: risk seeking for gains and risk aversion for losses of low probability, coupled with risk aversion for gains and risk seeking for losses of high probability. Choices consistent with this pattern have been observed in several studies, with and without monetary incentives[1] (Cohen, Jaffray, & Said, 1987; Fishburn & Kochenberger, 1979; Hershey & Schoemaker, 1980; Kahneman & Tversky, 1979; Payne, Laughhunn, & Crum, 1981; Wehrung, 1989). Risk seeking for low-probability gains may contribute to the popularity of gambling, whereas risk seeking for high-probability losses is consistent with the tendency to undertake risk in order to avoid a sure loss.

Because the fourfold pattern is observed for a wide range of payoffs, it cannot be explained by the shape of the utility function as proposed earlier by Friedman and Savage (1948) and by Markowitz (1952). Instead, it suggests a nonlinear transformation of the probability scale, first proposed by Preston and Baratta (1948) and further discussed by Edwards (1962) and others. This notion is one of the cornerstones of prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992), which provides the theoretical framework used in the present article. According to this theory, the value of a simple prospect that offers a probability p to win $\$x$ (and probability $1 - p$

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[1] Risk seeking for long shots was reported by Kachelmeier and Shehata (1992) in an experiment conducted in China with real payoffs that were considerably higher than the normal monthly incomes of the participants.

to win nothing) is given by $w(p)v(x)$, where v measures the subjective value of the outcome x, and w measures the impact of p on the desirability of the prospect. The values of w are called decision weights; they are normalized so that $w(0) = 0$, and $w(1) = 1$. It is important to note that w should not be interpreted as a measure of degree of belief. A decision maker may believe that the probability of heads on a toss of a coin is one-half but give this event a lower weight in the evaluation of a prospect.

According to prospect theory, the value function v and the weighting function w exhibit diminishing sensitivity: marginal impact diminishes with distance from a reference point. For monetary outcomes, the status quo generally serves as the reference point that distinguishes gains from losses. Thus, diminishing sensitivity gives rise to an S-shaped value function, with $v(0) = 0$, that is concave for gains and convex for losses. For probability, there are two natural reference points—certainty and impossibility—that correspond to the endpoints of the scale. Therefore, diminishing sensitivity implies that increasing the probability of winning a prize by .1 has more impact when it changes the probability of winning from .9 to 1.0 or from 0 to .1 than when it changes the probability from, say, .3 to .4 or from .6 to .7. This gives rise to a weighting function that is concave near zero and convex near one. Figure 1 depicts the weighting functions for gains and for losses, estimated from the median data of Tversky and Kahneman (1992).[2] Such a function overweights small probabilities and underweights moderate and high probabilities, which explains the fourfold pattern of risk attitudes illustrated in Table 1. It also accounts for the well-known certainty effect discovered by Allais (1953). For example, whereas most people prefer a sure \$30 to an 80% chance of winning \$45, most people also prefer a 20% chance of winning \$45 to a 25% chance of winning \$30, contrary to the substitution axiom of expected utility theory (Tversky & Kahneman, 1986). This observation is consistent with an S-shaped weighting function satisfying $w(.20)/w(.25) \geq w(.80)/w(1.0)$. Such a function appears to provide a unified account of a wide range of empirical findings (see Camerer & Ho, 1994).

A choice model that is based on a nonlinear transformation of the probability scale assumes that the decision maker knows the probabilities associated with the possible outcomes. With the notable exception of games of chance, however, these probabilities are unknown, or at least not specified in advance. People generally do not know the probabilities associated with events such as the guilt of a defendant, the outcome of a football game, or the future price of oil. Following Knight (1921), decision theorists distinguish between risky (or chance) prospects where the probabilities associated with outcomes are assumed to be known, and uncertain prospects where these probabilities are not assumed to be known. To describe individual choice between uncertain prospects, we need to generalize the weighting function from risk to uncertainty. When the probabilities are unknown, however, we cannot describe decision weights as a simple transformation of the probability scale. Thus, we cannot plot the weighting function as we did in Figure 1, nor can we speak about the overweighting of low probabilities and underweighting of high probabilities.

Table 1
The Fourfold Pattern of Risk Attitudes

Probability	Gain	Loss
Low	C(\$100, .05) = \$14 (risk seeking)	C(−\$100, .05) = −\$8 (risk aversion)
High	C(\$100, .95) = \$78 (risk aversion)	C(−\$100, .95) = −\$84 (risk seeking)

Note. C is the median certainty equivalent of the prospect in question.

This article extends the preceding analysis from risk to uncertainty. To accomplish this, we first generalize the weighting function and introduce the principle of bounded subadditivity. We next describe a series of studies that demonstrates this principle for both risk and uncertainty, and we show that it is more pronounced for uncertainty than for risk. Finally, we discuss the relationship between decision weights and judged probabilities, and the role of ambiguity in choice under uncertainty. An axiomatic treatment of these concepts is presented in Tversky and Wakker (in press).

Theory

Let S be a set whose elements are interpreted as states of the world. Subsets of S are called *events*. Thus, S corresponds to the certain event, and ϕ is the null event. A weighting function W (on S) is a mapping that assigns to each event in S a number between 0 and 1 such that $W(\phi) = 0$, $W(S) = 1$, and $W(A) \geq W(B)$ if $A \supset B$. Such a function is also called a *capacity*, or a nonadditive probability.

As in the case of risk, we focus on simple prospects of the form (x, A), which offer \$$x$ if an uncertain event A occurs and nothing if A does not occur. According to prospect theory, the value of such a prospect is $W(A)v(x)$, where W is the decision weight associated with the uncertain event A. (We use W for uncertainty and w for risk.) Because the present treatment is confined to simple prospects with a single positive outcome, it is consistent with both the original and the cumulative versions of prospect theory (Tversky & Kahneman, 1992). It is consistent with expected utility theory if and only if W is additive, that is, $W(A \cup B) = W(A) + W(B)$ whenever $A \cap B = \phi$.[3]

Prospect theory assumes that W satisfies two conditions.

(i) Lower subadditivity: $W(A) \geq W(A \cup B) - W(B)$, provided A and B are disjoint and $W(A \cup B)$ is bounded away from one.[4] This inequality captures the possibility effect: The impact of an event A is greater when it is added to the null event than when it is added to some nonnull event B.

(ii) Upper subadditivity: $W(S) - W(S - A) \geq W(A \cup B) - W(B)$, provided A and B are disjoint and $W(B)$ is bounded

[2] Figure 1 corrects a minor error in the original drawing.

[3] For other discussions of decision weights for uncertain events, see Hogarth and Einhorn (1990), Viscusi (1989), and Wakker (1994).

[4] The boundary conditions are needed to ensure that we always compare an interval that includes an endpoint to an interval that is bounded away from the other endpoint (see Tversky & Wakker, in press, for a more rigorous formulation).

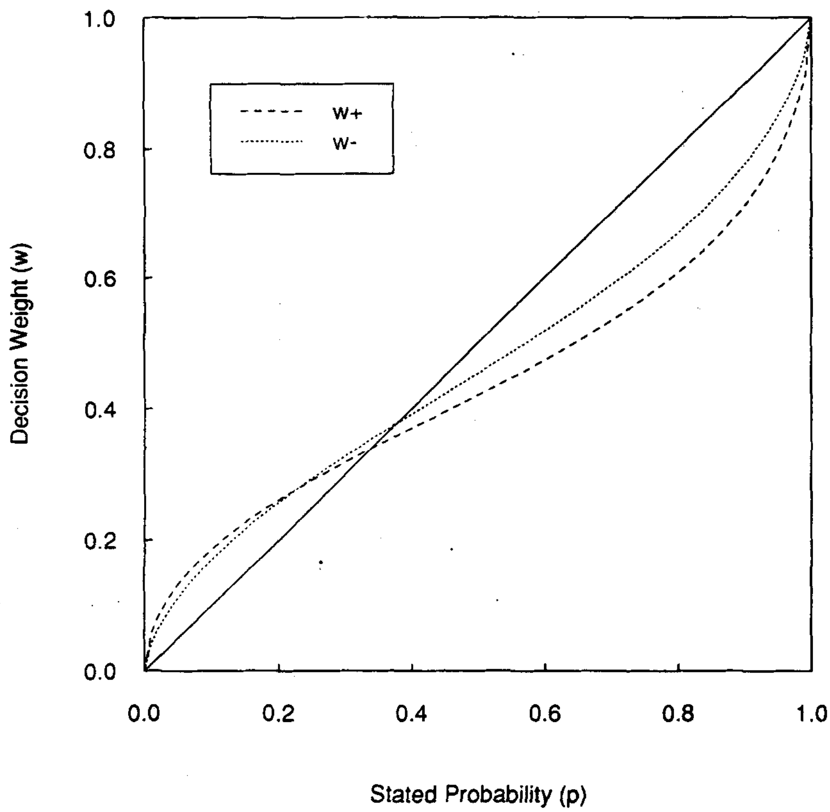

Figure 1. Weighting functions for gains (w+) and losses (w−).

away from zero.[5] This inequality captures the certainty effect: The impact of an event A is greater when it is subtracted from the certain event S than when it is subtracted from some uncertain event $A \cup B$.

A weighting function W satisfies bounded subadditivity, or subadditivity (SA) for short, if it satisfies both (i) and (ii) above. According to such a weighting function, an event has greater impact when it turns impossibility into possibility or possibility into certainty than when it merely makes a possibility more or less likely. To illustrate, consider the possible outcome of a football game. Let H denote the event that the home team wins the game, V denote the event that the visiting team wins, and T denote a tie. Hence, $S = H \cup V \cup T$. Lower SA implies that $W(T)$ exceeds $W(H \cup T) - W(H)$, whereas upper SA implies that $W(H \cup V \cup T) - W(H \cup V)$ exceeds $W(H \cup T) - W(H)$. Thus, adding the event T (a tie) to ϕ has more impact than adding T to H, and subtracting T from S has more impact than subtracting T from $H \cup T$. These conditions extend to uncertainty the principle that increasing the probability of winning a prize from 0 to p has more impact than increasing the probability of winning from q to $q + p$, and decreasing the probability of winning from 1 to $1 - p$ has more impact than decreasing the probability of winning from $q + p$ to q. To investigate these properties empirically, consider four simple prospects, each of which offers a fixed prize if a particular event (H, T, $H \cup V$, or $H \cup T$) occurs and nothing if it does not. By asking people to price these prospects, we can estimate the decision weights associated with the respective events and test both lower and upper SA, provided the value function is scaled independently.

Several comments concerning this analysis are in order. First, risk can be viewed as a special case of uncertainty where probability is defined through a standard chance device so that the probabilities of outcomes are known. Under this interpretation, the S-shaped weighting function of Figure 1 satisfies both lower and upper SA. Second, we have defined these properties in terms of the weighting function W that is not directly observable but can be derived from preferences (see Wakker & Tversky, 1993). Necessary and sufficient conditions for bounded SA in terms of the observed preference order are presented by Tversky and Wakker (in press) in the context of cumulative prospect theory. Third, the concept of bounded SA is more general than the property of diminishing sensitivity, which gives rise to a weighting function that is concave for relatively unlikely events and convex for relatively likely events. Finally, there is evidence to suggest that the decision weights for complementary events typically sum to less than one, that is, $W(A) + W(S - A) \leq 1$ or equivalently, $W(A) \leq W(S) - W(S - A)$. This property,

[5] The upper subadditivity of W is equivalent to the lower subadditivity of the dual function $W'(A) = 1 - W(S - A)$.

Table 2
A Demonstration of Subadditivity in Betting on the Outcome of a Stanford–Berkeley Football Game

		Events				
Problem	Option	A	B	C	D	Preference (%)
1	f_1	\$25	0	0	0	61
	g_1	0	0	\$10	\$10	39
2	f_2	0	0	0	\$25	66
	g_2	\$10	\$10	0	0	34
3	f_3	\$25	0	0	\$25	29
	g_3	\$10	\$10	\$10	\$10	71

Note. A = Stanford wins by 7 or more points; B = Stanford wins by less than 7 points; C = Berkeley ties or wins by less than 7 points; D = Berkeley wins by 7 or more points. Preference = percentage of respondents (N = 112) that chose each option.

called *subcertainty* (Kahneman & Tversky, 1979), can also be interpreted as evidence that upper SA has more impact than lower SA; in other words, the certainty effect is more pronounced than the possibility effect. Some data consistent with this property are presented below.

An Illustration

We next present an illustration of SA that yields a new violation of expected utility theory. We asked 112 Stanford students to choose between prospects defined by the outcome of an upcoming football game between Stanford and the University of California at Berkeley. Each participant was presented with three pairs of prospects, displayed in Table 2. The percentage of respondents who chose each prospect appears on the right. Half of the participants received the problems in the order presented in the table; the other half received the problems in the opposite order. Because we found no significant order effects, the data were pooled. Participants were promised that 10% of all respondents, selected at random, would be paid according to one of their choices.

Table 2 shows that, overall, f_1 was chosen over g_1, f_2 over g_2, and g_3 over f_3. Furthermore, the triple (f_1, f_2, g_3) was the single most common pattern, selected by 36% of the respondents. This pattern violates expected utility theory, which implies that a person who chooses f_1 over g_1 and f_2 over g_2 should also choose f_3 over g_3. However, 64% of the 55 participants who chose f_1 and f_2 in Problems 1 and 2 chose g_3 in Problem 3, contrary to expected utility theory. This pattern, however, is consistent with the present account. To demonstrate, we apply prospect theory to the modal choices in Table 2. The choice of f_1 over g_1 in Problem 1 implies that

1
$$v(25)W(A) > v(10)W(C \cup D).$$

Similarly, the choice of f_2 over g_2 in Problem 2 implies that

2
$$v(25)W(D) > v(10)W(A \cup B).$$

Adding the two inequalities and rearranging terms yields

3
$$\frac{W(A) + W(D)}{W(A \cup B) + W(C \cup D)} > \frac{v(10)}{v(25)}.$$

On the other hand, the choice of g_3 over f_3 in Problem 3 implies that

$$v(10)W(A \cup B \cup C \cup D) > v(25)W(A \cup D),$$

4
$$\text{or} \quad \frac{v(10)}{v(25)} > \frac{W(A \cup D)}{W(A \cup B \cup C \cup D)}.$$

Consequently, the modal choices imply

5
$$\frac{W(A) + W(D)}{W(A \cup B) + W(C \cup D)} > \frac{W(A \cup D)}{W(A \cup B \cup C \cup D)}.$$

It can be shown that this inequality is consistent with a subadditive weighting function. Moreover, the inequality follows from such a weighting function, provided that subcertainty holds. To demonstrate, note that according to lower SA, $W(A) + W(D) \geq W(A \cup D)$. Furthermore, it follows from subcertainty that

6
$$W(A \cup B) + W(C \cup D) \leq W(A \cup B \cup C \cup D) = 1.$$

Thus, the left-hand ratio exceeds the right-hand ratio, in accord with the modal choices. Note that under expected utility theory W is an additive probability measure, hence the left-hand ratio and the right-hand ratio must be equal.

Relative Sensitivity

As noted earlier, prospect theory assumes SA for both risk and uncertainty. We next propose that this effect is stronger for uncertainty than for risk. In other words, both lower and upper SA are amplified when outcome probabilities are not specified.

To test this hypothesis, we need a method for comparing different domains or sources of uncertainty (e.g., the outcome of a football game or the spin of a roulette wheel). Consider two sources, **A** and **B**, and suppose that the decision weights for both sources satisfy bounded subadditivity. We say that the decision maker is less sensitive to **B** than to **A** if the following two conditions hold for all disjoint events A_1, A_2 in **A**, and B_1, B_2 in **B**, provided all values of W are bounded away from 0 and 1.

7
$$\text{If } W(B_1) = W(A_1) \text{ and } W(B_2) = W(A_2)$$
$$\text{then } W(B_1 \cup B_2) \leq W(A_1 \cup A_2). \quad (1)$$

8
$$\text{If } W(S - B_1) = W(S - A_1) \text{ and } W(S - B_2) = W(S - A_2),$$
$$\text{then } W(S - [B_1 \cup B_2]) \geq W(S - [A_1 \cup A_2]). \quad (2)$$

The first condition says that the union of disjoint events from **B** "loses" more than the union of matched events from **A**. The second condition imposes the analogous requirement on the dual function. Thus, a person is less sensitive[6] to **B** than to **A** if **B** produces more lower SA and more upper SA than does **A**.

This definition can be readily stated in terms of preferences.

[6] Relative sensitivity is closely related to the concept of relative curvature for subjective dimensions introduced by Krantz and Tversky (1975).

Table 3
Outline of Studies

	Study 1	Study 2	Study 3
Participants	NBA fans ($N = 27$)	NFL fans ($N = 40$)	Psychology students ($N = 45$)
Sources	Chance NBA playoffs San Francisco temperature	Chance Super Bowl Dow–Jones	Chance San Francisco temperature Beijing temperature

Note. NBA = National Basketball Association; NFL = National Football League.

To illustrate, consider a comparison between uncertainty and chance.[7] Suppose B_1 and B_2 are disjoint uncertain events (e.g., the home team wins or the home team ties a particular football game). Let A_1 and A_2 denote disjoint chance events (e.g., a roulette wheel landing red or landing green). The hypothesis that people are less sensitive to the uncertain source **B** than to the chance source **A** implies the following preference condition. If one is indifferent between receiving \$50 if the home team wins the game or if a roulette wheel lands red ($p = 18/38$), and if one is also indifferent between receiving \$50 if the home team ties the game or if a roulette wheel lands green (i.e., zero or double zero, $p = 2/38$), then one should prefer receiving \$50 if a roulette wheel lands either green or red ($p = 20/38$) to receiving \$50 if the home team either wins or ties the game.

The following studies test the two hypotheses discussed above. First, decision makers exhibit bounded subadditivity under both risk and uncertainty. Second, decision makers are generally less sensitive to uncertainty than to risk.

Experimental Tests

We conducted three studies using a common experimental paradigm. On each trial, participants chose between an uncertain (or risky) prospect and various cash amounts. These data were used to estimate the certainty equivalents of each prospect (i.e., the sure amount that the participant considers as attractive as the prospect) and to derive decision weights. The basic features of the studies are outlined in Table 3.

Method

Participants. The participants in the first study were 27 male Stanford students (median age = 21) who responded to advertisements calling for basketball fans to take part in a study of decision making. Participants received \$15 for participating in two 1-hour sessions, spaced a few days apart. The participants in the second study were 40 male football fans (median age = 21), recruited in a similar manner. They were promised that in addition to receiving \$15 for their participation in two 1-hour sessions, some of them would be selected at random to play one of their choices for real money. The participants in the third study were 45 Stanford students enrolled in an introductory psychology course (28 men, 17 women, median age = 20) who took part in a 1-hour session for course credit. The responses of a few additional participants (one from Study 1, four from Study 2, and three from Study 3) were excluded from the analysis because they exhibited a great deal of internal inconsistency. We also excluded a very small number of responses that were completely out of line with an individual's other responses.

Procedure. The experiment was run using a computer. Each trial involved a series of choices between a prospect that offered a prize contingent on chance or an uncertain event (e.g., a 25% chance to win a prize of \$150) and a descending series of sure payments (e.g., receive \$40 for sure). In Study 1, the prize was always \$75 for half the respondents and \$150 for the other half; in Studies 2 and 3, the prize for all respondents was \$150. Certainty equivalents were inferred from two rounds of such choices. The first round consisted of six choices between the prospect and sure payments, spaced roughly evenly between \$0 and the prize amount. After completing the first round of choices, a new set of seven sure payments was presented, spanning the narrower range between the lowest payment that the respondent had accepted and highest payment that the respondent had rejected. The program enforced internal consistency. For example, no respondent was allowed to prefer \$30 for sure over a prospect and also prefer the same prospect over a sure \$40. The program allowed respondents to backtrack if they felt they had made a mistake in the previous round of choices.

The certainty equivalent of each prospect was determined by a linear interpolation between the lowest value accepted and the highest value rejected in the second round of choices. This interpolation yielded a margin of error of ±\$2.50 for the \$150 prospects and ±\$1.25 for the \$75 prospects. We wish to emphasize that although our analysis is based on certainty equivalents, the data consisted of a series of choices between a given prospect and sure outcomes. Thus, respondents were not asked to generate certainty equivalents; instead, these values were inferred from choices.

Each session began with detailed instructions and practice. In Study 1, the first session consisted of chance prospects followed by basketball prospects; the second session replicated the chance prospects followed by prospects defined by a future temperature in San Francisco. In Study 2, the first session consisted of chance prospects followed by Super Bowl prospects; the second session replicated the chance prospects followed by prospects defined by a future value of the Dow–Jones index. Study 3 consisted of a single session in which the chance prospects were followed by prospects defined by a future temperature in San Francisco and Beijing; the order of the latter two sources was counterbalanced. The order of the prospects within each source was randomized.

Sources of uncertainty. Chance prospects were described in terms of a random draw of a single poker chip from an urn containing 100 chips numbered consecutively from 1 to 100. Nineteen prospects of the form (x, p) were constructed where p varied from .05 to .95 in multiples of .05. For example, a typical chance prospect would pay \$150 if the number of the poker chip is between 1 and 25, and nothing otherwise. This design yields 90 tests of lower SA and 90 tests of upper SA for each participant.

[7] Although probabilities could be generated by various chance devices, we do not distinguish between them here, and treat risk or chance as a single source of uncertainty.

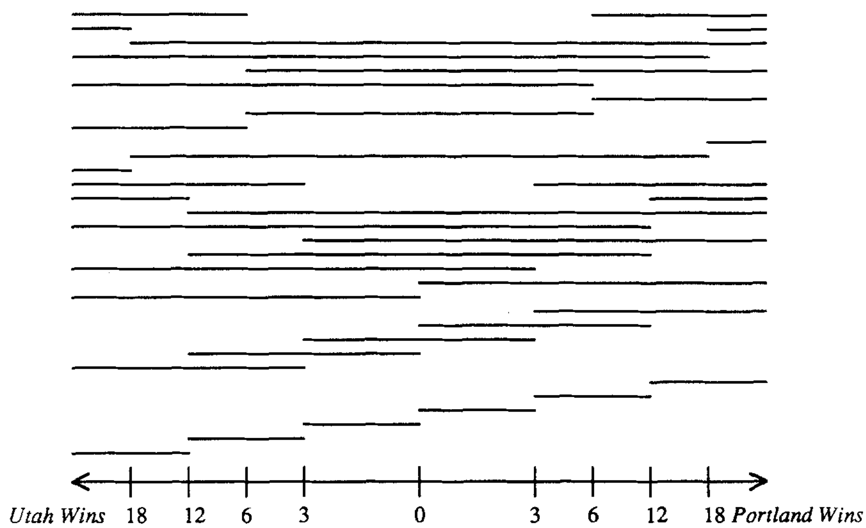

Figure 2. Event space for prospects defined by the result of the Utah–Portland basketball game. The horizontal axis refers to the point spread in that game. Each row denotes a target event that defines a prospect used in Study 1. Segments that extend up to the arrowhead represent unbounded intervals. Each interval includes the more extreme endpoint relative to 0, but not the less extreme endpoint.

Basketball prospects were defined by the result of the first game of the 1991 National Basketball Association (NBA) quarter final series between the Portland Trailblazers and the Utah Jazz. For example, a typical prospect would pay \$150 if Portland beats Utah by more than 6 points. The event space is depicted in Figure 2. Each of the 32 rows in the figure represents a target event A that defines an uncertain prospect (x, A). For example, the top row in Figure 2, which consists of two segments, represents the event "the margin of victory exceeds 6 points." This design yields 28 tests of lower SA and 12 tests of upper SA. For example, one test of lower SA is obtained by comparing the decision weight for the event "Utah wins" to the sum of the decision weights for the two events "Utah wins by up to 12 points" and "Utah wins by more than 12 points."

Super Bowl prospects were defined by the result of the 1992 Super Bowl game between the Buffalo Bills and the Washington Redskins. The event space is depicted in Figure 3. It includes 28 target events yielding 30 tests of lower SA and 17 tests of upper SA.

Dow-Jones prospects were defined by the change in the Dow-Jones Industrial Average over the subsequent week. For example, a typical prospect would pay \$150 if the Dow-Jones goes up by more than 50 points over the next seven days. The event space has the same structure as that of the Super Bowl (Figure 3).

San Francisco temperature prospects were defined by the daytime high temperature in San Francisco on a given future date. The 20 target events used in Studies 1 and 3 are depicted in Figure 4. This design yields 30 tests of lower SA and 10 tests of upper SA. For example, a typical prospect would pay \$75 if the daytime high temperature in downtown San Francisco on April 1, 1992, is between 65° and 80°. Similarly, Beijing temperature prospects were defined by the daytime high temperature in Beijing on a given future day. The event space is identical to the San Francisco temperature in Study 3, as depicted in Figure 4.

Results

To test lower and upper SA, the decision weights for each respondent were derived as follows. Using the choice data, we first estimated the certainty equivalent C of each prospect by linear interpolation, as described earlier. According to prospect theory, if $C(x, A) = y$, then $v(y) = W(A)v(x)$ and $W(A) = v(y)/v(x)$. The decision weight associated with an uncertain event A, therefore, can be computed if the value function v for gains is known. Previous studies (e.g., Tversky, 1967) have indicated that the value function for gains can be approximated by a power function of the form $v(x) = x^\alpha$, $0 \le \alpha \le 1$. This form is characterized by the assumption that multiplying the prize of a prospect by a positive constant multiplies its certainty equivalent by the same constant.[8] This prediction was tested using the data from Study 1 in which each event was paired both with a prize of \$75 and with a prize of \$150. Consistent with a power value function, we found no significant difference between $C(150, A)$ and $2C(75, A)$ for any of the sources.

Although the present data are consistent with a power function, the value of the exponent cannot be estimated from simple prospects because the exponent α can be absorbed into W. To estimate the exponent for gains, we need prospects with two positive outcomes. Such prospects were investigated by Tversky and Kahneman (1992), using the same experimental procedure and a similar subject population. They found that estimates of the exponent did not vary markedly across respondents and the median estimate of the exponent was .88. In the analysis that follows, we first assume a power value function with an exponent of .88 and test lower and upper SA using this function. We then show that the test of SA is robust with respect to substantial variations in the exponent. Further analyses are

[8] This follows from the fact that for $t > 0$, the value of the prospect (tx, A) is $W(A)(tx)^\alpha$; hence, $C(tx, A) = W(A)^{1/\alpha}tx$, which equals $tC(x, A)$.

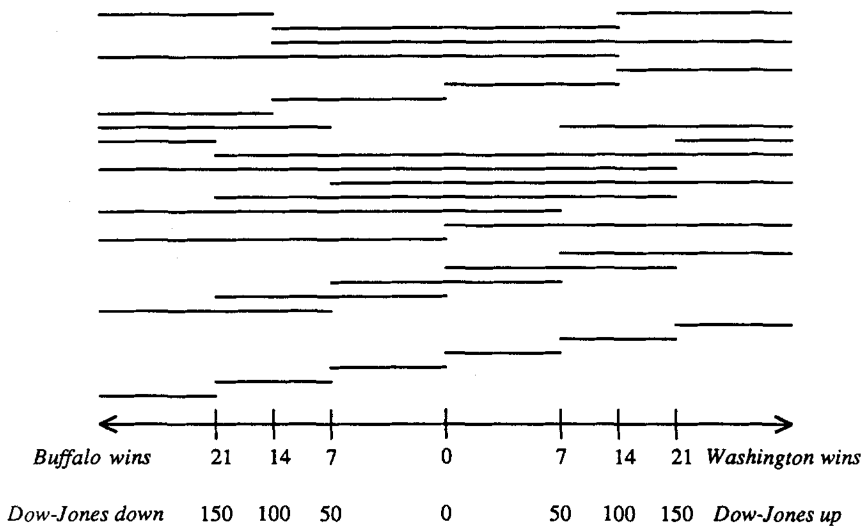

Figure 3. Event space for prospects defined by the result of the Super Bowl game between Washington and Buffalo (and for the Dow-Jones prospects). The horizontal axis refers to the point spread in the Super Bowl (and the change in the Dow-Jones in the next week). Each row denotes a target event that defines a prospect used in Study 2. Segments that extend up to the arrowhead represent unbounded intervals. Each interval includes the more extreme endpoint relative to 0, but not the less extreme endpoint.

based on an ordinal method that makes no assumption about the functional form of v.

Using the estimated W for each source of uncertainty, we define measures of the degree of lower and upper SA as follows. Recall that lower SA requires that $W(A) \geq W(A \cup B) - W(B)$, for $A \cap B = \phi$. Hence, the difference between the two sides of the inequality,

9
$$D(A, B) \equiv W(A) + W(B) - W(A \cup B),$$

provides a measure of the degree of lower SA. Similarly, recall that upper SA requires that $1 - W(S - A) \geq W(A \cup B) - W(B)$, for $A \cap B = \phi$. Hence, the difference between the two sides of the inequality,

10
$$D'(A, B) \equiv 1 - W(S - A) - W(A \cup B) + W(B),$$

provides a measure of the degree of upper SA.

Table 4 presents the overall proportion of tests, across participants, that strictly satisfy lower and upper SA (i.e., $D > 0$, $D' > 0$) for each source of uncertainty. Note that if W were additive (as implied by expected utility theory), then both D and D' are

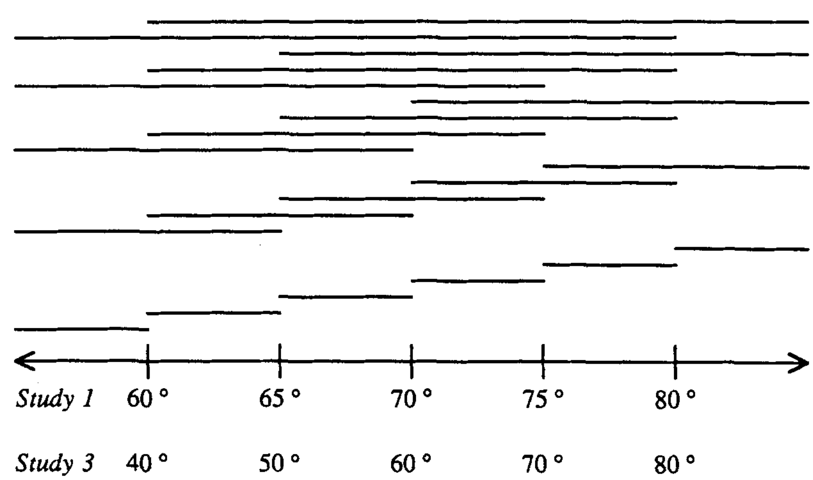

Figure 4. Event space for prospects defined by future temperatures in San Francisco and Beijing. The horizontal axis refers to the daytime high temperature on a given date. Each row denotes a target event that defines a prospect used in Studies 1 and 3. Segments that extend up to the arrowhead represent unbounded intervals. Each interval includes the left endpoint but not the right endpoint.

Table 4
Proportion of Tests That Strictly Satisfy Lower and Upper Subadditivity (SA)

Source	Study 1 Lower SA	Study 1 Upper SA	Study 2 Lower SA	Study 2 Upper SA	Study 3 Lower SA	Study 3 Upper SA
Chance	.80	.81	.77	.83	.81	.79
Basketball	.88	.83				
Super Bowl			.86	.87		
Dow-Jones			.77	.87		
S.F. temp.	.83	.89			.85	.89
Beijing temp.					.89	.91

Note. S.F. = San Francisco; temp. = temperature.

expected to be zero; hence, all entries in Table 4 should be close to one-half. However, each entry in Table 4 is significantly greater than one-half ($p < .01$, by a binomial test), as implied by SA.

To obtain global measures of lower and upper SA, let d and d', respectively, be the mean values of D and D' for a given respondent. Besides serving as summary statistics, these indexes have a simple geometric interpretation if the risky weighting function is roughly linear except near the endpoints. It is easy to verify that within the linear portion of the graph, D and D' do not depend on A and B, and the summary measures d and d' correspond to the "lower" and "upper" intercepts of the weighting function (see Figure 5). Its slope, $s = 1 - d - d'$, can then be interpreted as a measure of sensitivity to probability changes.

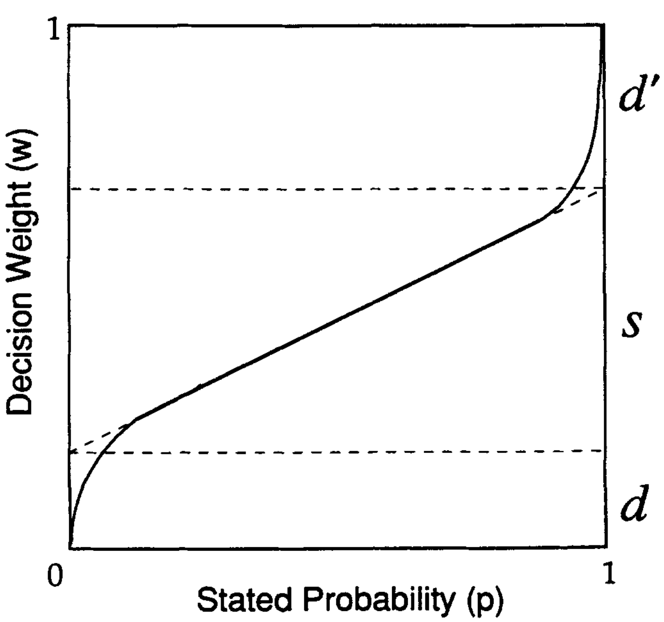

Figure 5. A weighting function that is linear except near the endpoints (d = "lower" intercept of the weighting function; d' = "upper" intercept of the weighting function; s = slope).

For uncertainty, we cannot plot d and d' as in Figure 5. However, d and d' have an analogous interpretation as a "possibility gap" and "certainty gap," respectively, if W is roughly linear except near the endpoints.[9] Note that under expected utility theory, $d = d' = 0$ and $s = 1$, whereas prospect theory implies $d \geq 0$, $d' \geq 0$, and $s \leq 1$. Thus, prospect theory implies less sensitivity to changes in uncertainty than is required by expected utility theory. To test these predictions, we computed the values of d, d', and s, separately for each respondent. Table 5 presents the median values of these indexes, across respondent for each source of uncertainty.

In accord with SA, each value of d and d' in Table 5 is significantly greater than zero ($p < .05$). Furthermore, both indexes are larger for uncertainty than for chance: The mean values of d and of d' for the uncertain sources are significantly greater than each of the corresponding indexes for chance ($p < .01$, separately for each study). Finally, consistent with subcertainty, d' tends to exceed d, though this difference is statistically significant only in Studies 2 and 3 ($p < .05$).

Recall that all participants evaluated the same set of risky prospects, and that respondents in each of the three studies evaluated two different types of uncertain prospects (see Table 3). Figure 6 plots, for each respondent, the average sensitivity measure s for the two uncertain sources against s for the risky source. (One respondent who produced a negative s was excluded from this analysis.) These data may be summarized as follows. First, all values of s for the uncertain prospects and all but two values of s for the risky prospects were less than or equal to one as implied by SA. Second, the values of s are considerably higher for risk (mean $s = .74$) than for uncertainty (mean $s = .53$), as demonstrated by the fact that 94 out of 111 points lie below the identity line ($p < .01$ by a sign test). Third, the data reveal a significant correlation between the sensitivity measures for risk and for uncertainty ($r = .37$, $p < .01$). The average correlation between the uncertain sources is .40. If we restrict the analysis to Studies 1 and 2 that yielded more stable data (in part because the risky prospects were replicated), the correlation between sensitivity for risk and for uncertainty increases to .51, and the mean correlation between the uncertain sources increases to .54. These correlations indicate the presence of consistent individual differences in SA and suggest that sensitivity to uncertainty is an important attribute that distinguishes among decision makers. An axiomatic analysis of the conditions under which one individual is consistently more SA than another is presented in Tversky and Wakker (in press).

Robustness. The preceding analysis summarized in Table 5 assumes a power value function with an exponent $\alpha = .88$. To investigate whether the above conclusions depend on the particular choice of the exponent, we reanalyzed the data using different values of α varying from one-half to one. To appreciate the impact of this difference, consider the prospect that offers a one-third chance to win \$100. [We choose one-third because, according to Figure 1, $w(1/3)$ is approximately one-third.] The certainty equivalent of this prospect is \$33.33 if $\alpha = 1$, but it is

[9] More formally, this holds when $W(A \cup B) - W(A)$ does not depend on A, for all $A \cap B = \phi$, provided $W(A)$ is not too close to zero and $W(A \cup B)$ is not too close to 1.

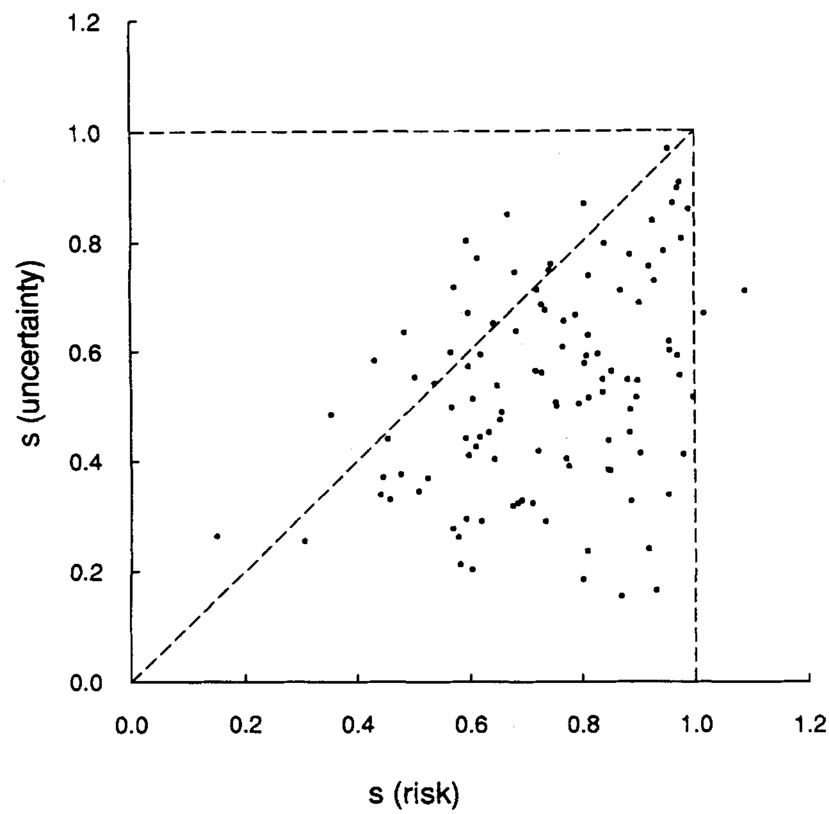

Figure 6. Joint distribution for all respondents of the sensitivity measure s for risk and uncertainty.

only \$11.11 if $\alpha = .5$. Table 6 shows that as α decreases (indicating greater curvature), d increases and d' decreases. More important, however, both d and d' are positive throughout the range for all sources, and the values of s are significantly smaller than one ($p < .01$) in all cases. SA, therefore, holds for a fairly wide range of variation in the curvature of the value function.

Ordinal analysis. The preceding analysis confirmed our hypothesis that people are less sensitive to uncertainty than to chance using the sensitivity measure s. We next turn to an ordinal test of this hypothesis that makes no assumptions about the value function. Let B_1, B_2 denote disjoint uncertain events, and let A_1, A_2 denote disjoint chance events. We searched among the responses of each participant for patterns satisfying

$$C(x, B_1) \geq C(x, A_1) \text{ and } C(x, B_2) \geq C(x, A_2) \quad \text{but } C(x, B_1 \cup B_2) < C(x, A_1 \cup A_2), \tag{3}$$

or

$$C(x, S - B_1) \leq C(x, S - A_1) \quad \text{and } C(x, S - B_2) \leq C(x, S - A_2) \quad \text{but } C(x, S - [B_1 \cup B_2]) > C(x, S - [A_1 \cup A_2]). \tag{4}$$

A response pattern that satisfies either condition 3 or 4 provides support for the hypothesis that the respondent is less sensitive to uncertainty (**B**) than to chance (**A**).

Several comments regarding this test are in order. First, note that if we replace the weak inequalities in conditions 3 and 4 with equalities, then these conditions reduce to the definition of relative sensitivity (see Equations 1 and 2). The above conditions are better suited for the present experimental design because participants were not asked to "match" intervals from different sources. Second, the present analysis is confined to contiguous intervals; conditions 3 and 4 may not hold when comparing contiguous to noncontiguous intervals (see Tversky

Table 5
Median Values of d, d', and s, Across Respondents, Measuring the Degree of Lower and Upper Subadditivity (SA) and Global Sensitivity, Respectively

	Study 1			Study 2			Study 3		
Source	d	d'	s	d	d'	s	d	d'	s
Chance	.06	.10	.81	.05	.19	.75	.11	.14	.72
Basketball	.21	.19	.61						
Super Bowl				.15	.23	.57			
Dow-Jones				.12	.22	.67			
S.F. temp.	.20	.26	.51				.27	.23	.50
Beijing temp.							.28	.32	.42

Note. S.F. = San Francisco; temp. = temperature.

& Koehler, 1994). Third, because of measurement error, the above conditions are not expected to hold for all comparisons; however, the conditions indicating less sensitivity to uncertainty than to chance are expected to be satisfied more frequently than the opposite conditions.

Let $M(\mathbf{B}, \mathbf{A})$ be the number of response patterns that satisfy condition 3 above (i.e., less sensitivity to uncertainty than to chance). Let $M(\mathbf{A}, \mathbf{B})$ be the number of response patterns that satisfy 3 with the As and Bs interchanged (i.e., less sensitivity to chance than to uncertainty). The ratio $m(\mathbf{B}, \mathbf{A}) = M(\mathbf{B}, \mathbf{A})/(M(\mathbf{B}, \mathbf{A}) + M(\mathbf{A}, \mathbf{B}))$ provides a measure of the degree to which a respondent is less sensitive to uncertainty than to chance, in the sense of condition 3. We define $M'(\mathbf{B}, \mathbf{A})$, $M'(\mathbf{A}, \mathbf{B})$, and $m'(\mathbf{B}, \mathbf{A})$ similarly for preference patterns that satisfy condition 4. If the respondent is invariably less sensitive to **B** than to **A**, then the ratios $m(\mathbf{B}, \mathbf{A})$ and $m'(\mathbf{B}, \mathbf{A})$ should be close to one. On the other hand, if the respondent is not more sensitive to one source than to another, these ratios should be close to one-half. Table 7 presents the median ratios, across respondent, comparing each of the five uncertain sources to chance. As predicted, all entries in the table are significantly greater than one-half ($p < .05$, by t tests), indicating that people are generally less sensitive to uncertainty than to chance.

We conclude this section with a brief methodological discussion. We have attributed the findings of bounded subadditivity and lower sensitivity for uncertainty than for risk to basic psychological attitudes toward risk and uncertainty captured by the weighting function. Alternatively, one might be tempted to account for these findings by a statistical model that assumes that the assessment of certainty equivalents, and hence the estimation of decision weights, is subject to random error that is bounded by the endpoints of the outcome scale, because $C(x, A)$ must lie between 0 and x. Although bounded error could contribute to SA, this model cannot adequately account for the observed pattern of results. First, it cannot explain the subadditivity observed in simple choice experiments that do not involve (direct or indirect) assessment of certainty equivalents, such as the Stanford–Berkeley problem presented in Table 2. More extensive evidence for both lower and upper SA in simple choices between risky prospects is reported by Wu and Gonzalez (1994), who also found some support for the stronger hypothesis that w is concave for low probabilities and convex for moderate and high probabilities. Second, a statistical model cannot readily account for the result of the ordinal analysis reported above that respondents were less sensitive to uncertainty than to chance. Third, because a random error model implies a bias toward one-half, it cannot explain the observation that the decision weight of an event that is as likely as not to occur is generally less than one-half (see Figures 7, 8, and 9 below). Finally, it should be noted that subadditivity and differential sensitivity play an important role in the pricing of risky and uncertain prospects, regardless of whether these phenomena are driven primarily by psychological or by statistical factors.

Table 6
Median Values of d, d′, and s Across Respondents, Measuring the Degree of Lower Subadditivity, Upper Subadditivity, and Global Sensitivity, Respectively, for Several Values of α Between .5 and 1

Source and index	α = 0.500	0.625	0.750	0.875	1.00
Chance (Study 1)					
d	.29	.19	.12	.06	.01
d′	.02	.05	.07	.10	.12
s	.66	.73	.77	.81	.83
Chance (Study 2)					
d	.28	.18	.11	.05	.003
d′	.09	.12	.15	.19	.23
s	.66	.70	.74	.75	.75
Chance (Study 3)					
d	.33	.24	.17	.11	.06
d′	.05	.08	.11	.14	.17
s	.59	.65	.69	.72	.75
Basketball					
d	.40	.33	.26	.21	.16
d′	.10	.14	.17	.19	.22
s	.50	.56	.58	.61	.63
Super Bowl					
d	.36	.28	.20	.15	.11
d′	.15	.18	.20	.23	.25
s	.49	.54	.55	.57	.60
Dow–Jones					
d	.34	.25	.17	.11	.07
d′	.12	.15	.19	.22	.25
s	.54	.61	.64	.67	.70
SF temp (Study 1)					
d	.40	.32	.26	.20	.15
d′	.15	.18	.22	.26	.30
s	.42	.48	.49	.51	.52
SF temp (Study 3)					
d	.47	.39	.33	.27	.22
d′	.15	.18	.20	.23	.26
s	.39	.43	.48	.50	.52
Beijing temp					
d	.48	.40	.34	.28	.23
d′	.21	.25	.29	.32	.35
s	.33	.38	.42	.42	.43

Note. SF = San Francisco; temp = temperature.

Discussion

The final section of this article addresses three topics. First, we explore the relationship between decision weights and judged probabilities. Second, we investigate the presence of preferences for betting on particular sources of uncertainty. Finally, we discuss descriptive and normative implications of the present results.

Preference and Belief

The present account distinguishes between decision weights derived from preferences and degree of belief expressed by probability judgments. What is the relation between the judged probability, $P(A)$, of an uncertain event, A, and its associated decision weight $W(A)$? To investigate this problem, we asked respondents, after they completed the choice task, to assess the probabilities of all target events. Following the analysis of decision weights, we define measures of the degree of lower and upper SA in probability judgments as follows:

Table 8
Median Values of d, d′, and s, Across Respondents, That Measure the Degree of Lower and Upper Subadditivity, SA, and Global Sensitivity, Respectively, for Judged Probability

	Study 1			Study 2			Study 3		
Source	d	d'	s	d	d'	s	d	d'	s
Basketball	.08	.11	.74						
Super Bowl				.11	.08	.81			
Dow–Jones				.07	.08	.84			
S.F. temp.	.13	.16	.70				.29	.21	.51
Beijing temp.							.24	.25	.53

Note. SA = Subadditivity; S.F. = San Francisco; temp. = temperature; s = degree of global sensitivity.

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$$D(A, B) \equiv P(A) + P(B) - P(A \cup B),$$

$$D'(A, B) \equiv 1 - P(S - A) + P(B) - P(A \cup B),$$

provided $A \cap B = \phi$. Clearly, P is additive if and only if $D = D' = 0$ for all disjoint A, B in S. As before, let d and d' be the mean values of D and D', respectively, and let $s = 1 - d - d'$. Table 8, which is the analog of Table 5, presents the median values of d, d', and s, across respondents, for each of the five uncertain sources.

All values of d and d' in Table 8 are significantly greater than zero ($p < .05$), demonstrating both lower and upper SA for probability judgments. Comparing Table 8 and Table 5 reveals that the values of s for judged probabilities (overall mean .70) are greater than the corresponding uncertain decision weights (overall mean .55). Thus, probability judgments exhibit less SA than do uncertain decision weights. This finding is consistent with a two-stage process in which the decision maker first assesses the probability P of an uncertain event A, then transforms this value by the risky weighting function w. Thus, $W(A)$ may be approximated by $w[P(A)]$.

We illustrate this model using the median risky and uncertain decision weights derived from Study 2 (assuming $\alpha = .88$). In Figure 7 we plot decision weights for chance prospects as a function of stated (objective) probabilities. In Figures 8 and 9, respectively, we plot decision weights for Super Bowl prospects and for Dow–Jones prospects as functions of (median) judged probabilities. The comparison of these figures reveals that the data in Figures 8 and 9 are less orderly than those in Figure 7. This is not surprising because judged probability (unlike stated probability) is measured with error, and because the uncertain decision weights exhibit greater variability (both within and between subjects) than risky decision weights. However, the underlying relation between probability and decision weights is nearly identical in the three figures.[10] This is exactly what we would expect if the uncertain weighting function W is obtained by applying the risky weighting function w to judged probabilities.

The subadditivity of probability judgments reported in Table 8 is consistent with support theory[11] (Tversky & Koehler, 1994), according to which $P(A) + P(B) \geq P(A \cup B)$. The combination of the two-stage model (which is based on prospect theory) with an analysis of probability judgments (which is based on support theory) can therefore explain our main finding that decision weights are more subadditive for uncertainty than for chance. This model also implies that the decision weight associated with an uncertain event (e.g., an airplane accident) increases when its description is unpacked into its constituents (e.g., an airplane accident caused by mechanical failure, terrorism, human error, or acts of God; see Johnson, Hershey, Meszaros, & Kunreuther, 1993). Furthermore, this model predicts greater subadditivity, ceteris paribus, when $A \cup B$ is a contiguous interval (e.g., future temperature between 60° and 80°) than when $A \cup B$ is not a contiguous interval (e.g., future temperature less than 60° or more than 80°). A more detailed treatment of this model will be presented elsewhere.

Table 7
Ordinal Analysis of Differential Sensitivity

	Study 1		Study 2		Study 3	
Source comparison	m	m'	m	m'	m	m'
Basketball vs. chance	.85	.64				
Super Bowl vs. chance			.91	.89		
Dow–Jones vs. chance			.79	.76		
S.F. temp. vs. chance	.76	.93			.87	.87
Beijing temp. vs. chance					.83	.94

Note. Each entry corresponds to the median value, across respondents, of m and m' measuring the degree to which respondents are less sensitive to uncertainty than to chance. S.F. = San Francisco; temp. = temperature.

Source Preference

The finding that people are less sensitive to uncertainty than to risk should be distinguished from the observation of ambigu-

[10] The smooth curves in Figures 5 and 6 were obtained by fitting the parametric form $w(p) = \delta p^{\gamma}/(\delta p^{\gamma} + [1 - p]^{\gamma})$, used by Lattimore, Baker, and Witte (1992). It assumes that the relation between w and p is linear in a log odds metric. The estimated values of the parameters in Figures 7, 8, and 9, respectively, are .69, .69, and .72 for γ, and .77, .76, and .76 for δ.

[11] In this theory, $P(A) + P(S - A) = 1$; hence, the equations for lower and upper SA coincide.

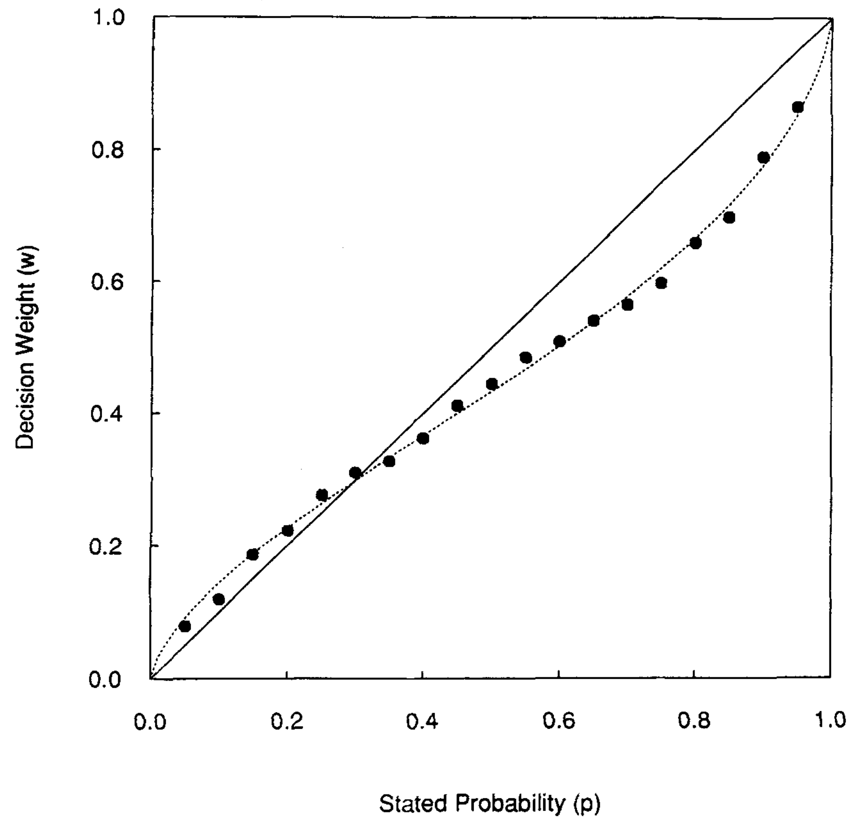

Figure 7. Median decision weights for chance prospects, from Study 2, plotted as a function of stated (objective) probabilities.

ity aversion: People often prefer to bet on known rather than unknown probabilities (Ellsberg, 1961). For example, people generally prefer to bet on either side of a fair coin than on either side of a coin with an unknown bias. These preferences violate expected utility theory because they imply that the sum of the subjective probabilities of heads and of tails is higher for the unbiased coin than for the coin with the unknown bias.

Recent research has documented some significant exceptions to ambiguity aversion. Heath and Tversky (1991) showed that people who were knowledgeable about sports but not about politics preferred to bet on sports events rather than on chance events that these people had judged equally probable. However, the same people preferred to bet on chance events rather than on political events that they had judged equally probable. Likewise, people who were knowledgeable about politics but not about sports exhibited the reverse pattern. These data support what Heath and Tversky call the competence hypothesis: People prefer to bet on their beliefs in situations where they feel competent or knowledgeable, and they prefer to bet on chance when they feel incompetent or ignorant. This account is consistent with the preference to bet on the fair rather than the biased coin, but it predicts additional preferences that are at odds with ambiguity aversion.

The preceding studies allow us to test the competence hypothesis against ambiguity aversion. Recall that the participants in Studies 1 and 2 were recruited for their knowledge of basketball and football, respectively. Ambiguity aversion implies a preference for chance over uncertainty because the probabilities associated with the sports events (e.g., Utah beating Portland) are necessarily vague or imprecise. In contrast, the competence hypothesis predicts that the sports fans will prefer to bet on the game than on chance.

To establish source preference, let **A** and **B** be two different sources of uncertainty. A decision maker is said to prefer source **A** to source **B** if for any events A in **A** and B in **B**. $W(A) = W(B)$ implies $W(S - A) > W(S - B)$, or equivalently, $C(x, A) = C(x, B)$ implies $C(x, S - A) > C(x, S - B)$, $x > 0$. To test for source preference we searched among the responses of each participants for patterns that satisfy $C(x, A) \geq C(x, B)$ and $C(x, S - A) > C(x, S - B)$. Thus, a decision maker who prefers to bet on event A than to bet on event B, and also prefers to bet against A than to bet against B exhibits a preference for source **A** over source **B**. The preference to bet on either side of a fair coin rather than on either side of a coin with an unknown bias illustrates such a preference for chance over uncertainty.

Let $K(\mathbf{A}, \mathbf{B})$ be the number of response patterns indicating a preference for source **A** over source **B**, as defined above, and let $K(\mathbf{B}, \mathbf{A})$ be the number of response patterns indicating the opposite preference. For each pair of sources, we computed the ratio $k(\mathbf{A}, \mathbf{B}) = K(\mathbf{A}, \mathbf{B})/(K(\mathbf{A}, \mathbf{B}) + K(\mathbf{B}, \mathbf{A}))$, separately for each respondent. This ratio provides a comparative index of source preference; it should equal one-half if neither source is

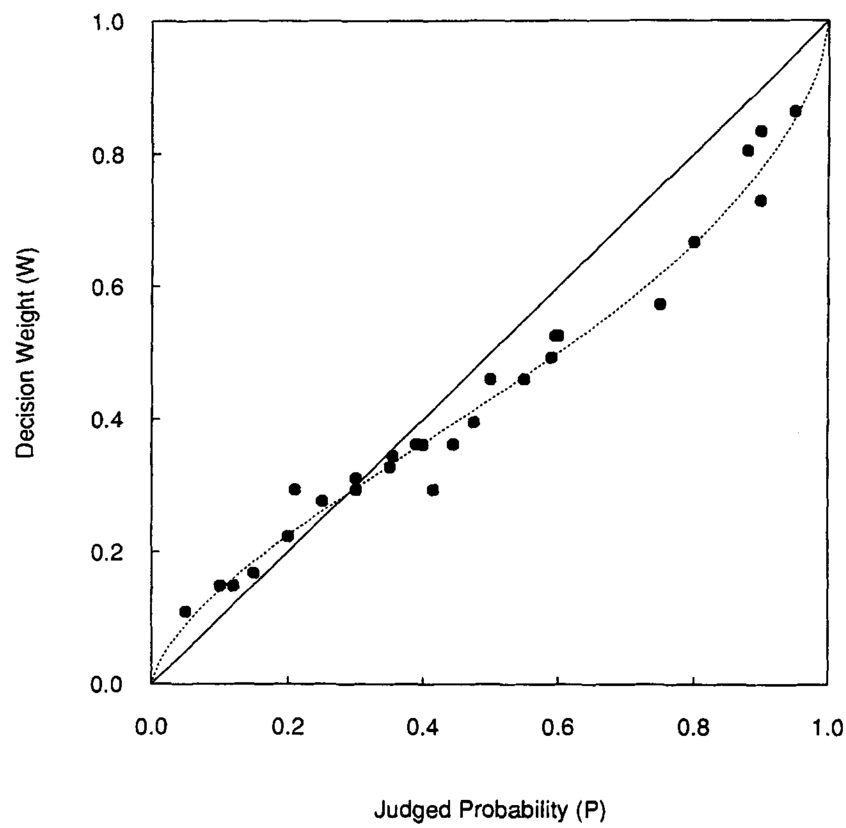

Figure 8. Median decision weights for Super Bowl prospects, from Study 2, plotted as a function of median judged probabilities.

preferred to the other, and it should be substantially greater than one-half if source A is generally preferred to source **B**.

The present data reveal significant source preferences that are consistent with the competence hypothesis but not with ambiguity aversion. In all three studies, participants preferred to bet on their uncertain beliefs in their area of competence rather than on known chance events. The basketball fans in Study 1 preferred betting on basketball than on chance (median $k = .76$, $p < .05$ by t test); the football fans in Study 2 preferred betting on the Super Bowl than on chance (median $k = .59$, though this effect is not statistically significant); and the students in Study 3 (who live near San Francisco) preferred betting on San Francisco temperature than on chance (median $k = .76$, $p < .01$). Two other comparisons consistent with the competence hypothesis are the preference for basketball over San Francisco temperature in Study 1 (median $k = .76$, $p < .05$), and the preference for San Francisco temperature over Beijing temperature in Study 3 (median $k = .86$, $p < .01$). For further discussions of ambiguity aversion and source preference, see Camerer and Weber (1992), Fox and Tversky (in press), and Frisch and Baron (1988).

Concluding Comments

Several authors (e.g., Ellsberg, 1961; Fellner, 1961; Keynes, 1921; Knight, 1921), critical of expected utility theory, distinguished among uncertain prospects according to the degree to which the uncertainty can be quantified. At one extreme, uncertainty is characterized by a known probability distribution; this is the domain of decision under risk. At the other extreme, decision makers are unable to quantify their uncertainty; this is the domain of decision under ignorance. Most decisions under uncertainty lie somewhere between these two extremes: People typically do not know the exact probabilities associated with the relevant outcomes, but they have some vague notion about their likelihood. The role of vagueness or ambiguity in decision under uncertainty has been the subject of much experimental and theoretical research.

In the present article we have investigated this issue using the conceptual framework of prospect theory. According to this theory, uncertainty is represented by a weighting function that satisfies bounded subadditivity. Thus, an event has more impact when it turns impossibility into possibility, or possibility into certainty, than when it merely makes a possibility more likely. This principle explains Allais's examples (i.e., the certainty effect) as well as the fourfold pattern of risk attitudes illustrated in Table 1. The experiments reported in this article demonstrate SA for both risk and uncertainty. They also show that this effect is more pronounced for uncertainty than for risk. The latter finding suggests the more general hypothesis that SA, and hence the departure from expected utility theory, is amplified by

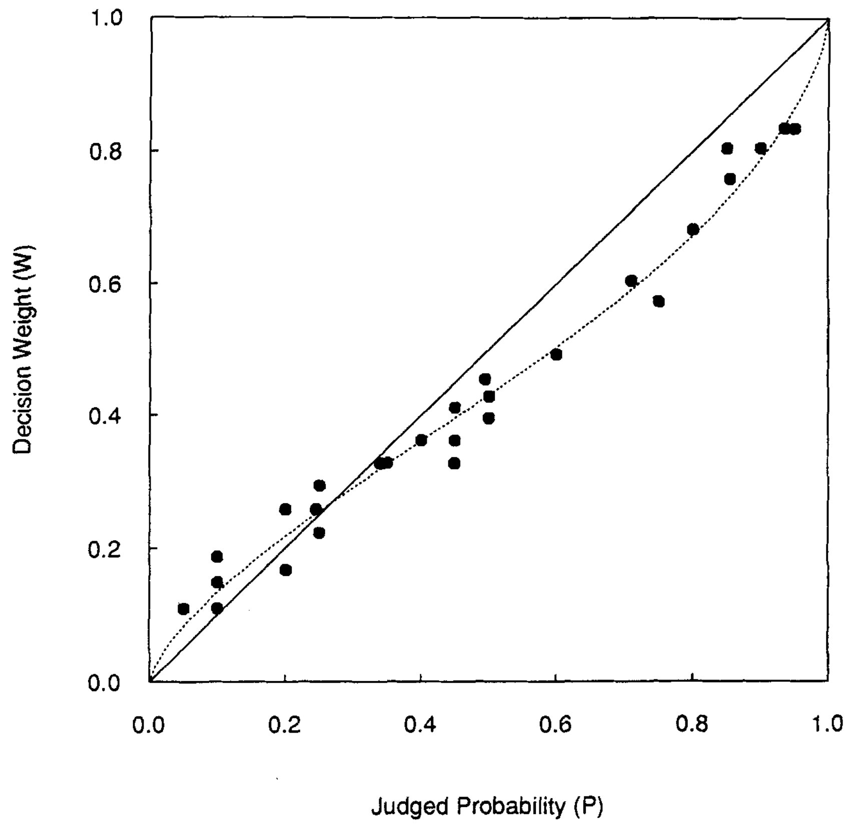

Figure 9. Median decision weights for Dow-Jones prospects, from Study 2, plotted as a function of median judged probability.

vagueness or ambiguity. Consequently, studies of decision under risk are likely to underestimate the degree of SA that characterizes decisions involving real-world uncertainty.[12] Subadditivity, therefore, emerges as a unifying principle of choice that is manifested to varying degrees in decisions under risk, uncertainty, and ignorance.

The psychological basis of bounded subadditivity includes both judgmental and preferential elements. As noted earlier, SA holds for judgments of probability (see Table 8), but it is more pronounced for decision weights (see Table 5). This amplification may reflect people's affective responses to positive and negative outcomes. Imagine owning a lottery ticket that offers some hope of winning a great fortune. Receiving a second ticket to the same lottery, we suggest, will increase one's hope of becoming rich but will not quite double it. The same pattern appears to hold for negative outcomes. Imagine waiting for the results of a biopsy. Receiving a preliminary indication that reduces the probability of malignancy by one-half, we suggest, will reduce fear by less than one-half. Thus, hope and fear seem to be subadditive in outcome probability. To the extent that the experience of hope and fear is treated as a consequence of an action, subadditivity may have some normative basis. If lottery tickets are purchased primarily for entertaining a fantasy, and protective action is undertaken largely to achieve peace of mind, then it is not unreasonable to value the first lottery ticket more than the second, and to value the elimination of a hazard more than a comparable reduction in its likelihood.

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[12] Evidence for substantial SA in the decisions of professional options traders is reported by Fox, Rogers, and Tversky (1995).

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