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Subjective Probability Assessment in Decision Analysis: Partition Dependence and Bias Toward the Ignorance Prior

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Decision and risk analysts have considerable discretion in designing procedures for eliciting subjective probabilities. One of the most popular approaches is to specify a particular set of exclusive and exhaustive events for which the assessor provides such judgments. We show that assessed probabilities are systematically biased toward a uniform distribution over all events into which the relevant state space happens to be partitioned, so that probabilities are "partition dependent." We surmise that a typical assessor begins with an "ignorance prior" distribution that assigns equal probabilities to all specified events, then adjusts those probabilities insufficiently to reflect his or her beliefs concerning how the likelihoods of the events differ. In five studies, we demonstrate partition dependence for both discrete events and continuous variables (Studies 1 and 2), show that the bias decreases with increased domain knowledge (Studies 3 and 4), and that top experts in decision analysis are susceptible to this bias (Study 5). We relate our work to previous research on the "pruning bias" in fault-tree assessment (e.g., Fischhoff et al. 1978) and show that previous explanations of pruning bias (enhanced availability of events that are explicitly specified, ambiguity in interpreting event categories, and demand effects) cannot fully account for partition dependence. We conclude by discussing implications for decision analysis practice.

Key words: probability assessment; risk assessment; subjective probability bias; fault tree
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1. Introduction

Decision and risk analysis models often require assessment of subjective probabilities for uncertain events, such as the failure of a dam or a rise in interest rates. Spetzler and Staël Von Holstein (1975) were the first to describe practical procedures for eliciting subjective probabilities from experts. Their procedures are still in use, largely unchanged, as reflected in work by Clemen and Reilly (2001), Cooke (1991), Keeney and von Winterfeldt (1991), Merkhofer (1987), and Morgan and Henrion (1990).

Human limitations of memory and information processing capacity often lead to subjective probabilities that are poorly calibrated or internally inconsistent, even when assessed by experts (see, e.g., Kahneman et al. 1982, Gilovich et al. 2002). In this paper, we study a particular bias in probability assessment that arises from the initial structuring of the elicitation. At this stage, the analyst, sometimes with the assistance of an expert, identifies relevant uncertainties and the specific events for which probabilities will be judged. Although existing probability assessment protocols provide guidance on important steps in the elicitation process (e.g., identifying and selecting experts, training experts in probability elicitation, and the probability assessment itself), little attention has been given to the choice of events to be assessed.

Analysts typically assume that the particular choice of events into which the state space is partitioned does not affect the assessed probability distribution over states. Unfortunately, our experimental results demonstrate that this assumption is unfounded: assessed probabilities can vary substantially with the partition that the analyst chooses. We refer to this phenomenon as *partition dependence* (see also Fox and Rottenstreich 2003). It is more general than the *pruning bias* documented in the assessment of fault trees by Fischhoff et al. (1978), in which particular causes of a system failure (e.g., reasons why a car might fail to start) are judged more likely when they are explicitly identified (e.g., dead battery or ignition system) than when pruned from the tree and relegated to a residual catchall category ("all other problems"). Most previous investigators have interpreted pruning bias as an availability or salience effect: when particular causes are singled out and made explicit rather than included implicitly in a catchall category, people are more likely to consider those causes in assessing probability; according to Fischhoff et al. (1978), "what is out of sight is also out of mind" (p. 333).

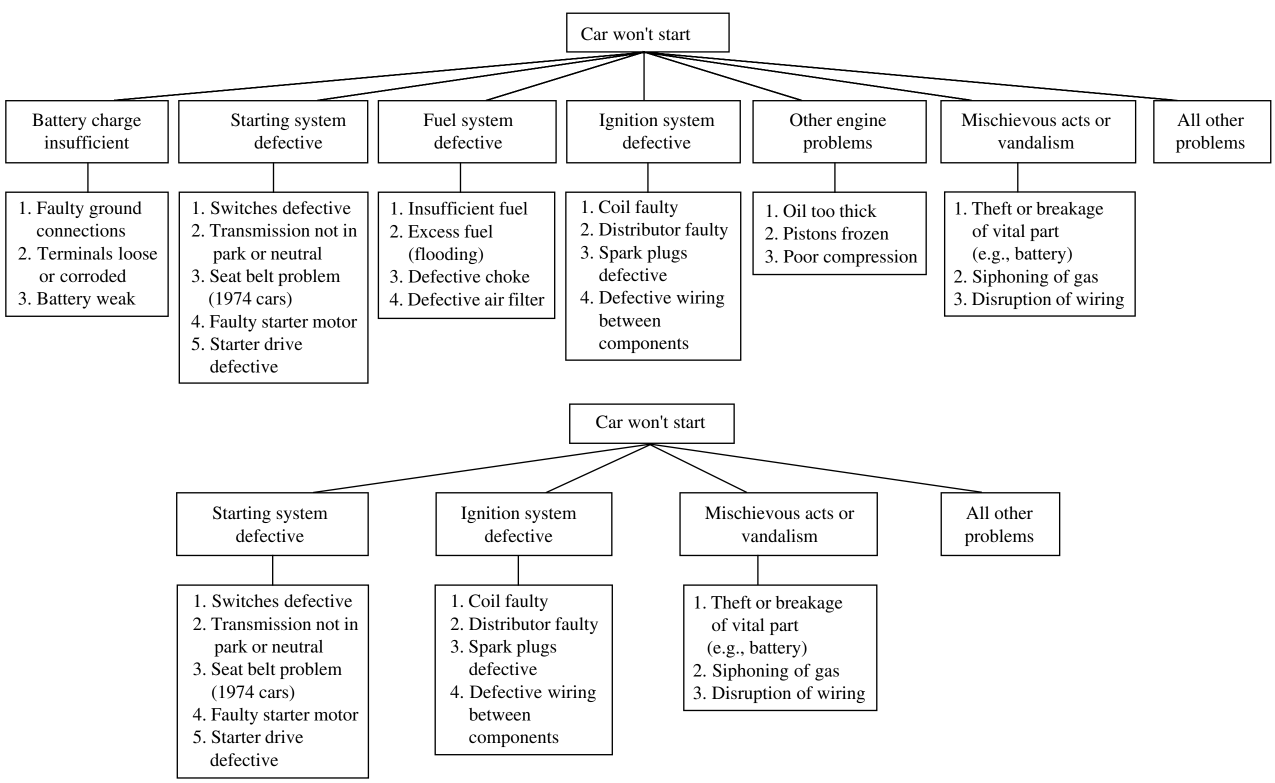
Our goal in this paper is to extend the investigation of pruning bias from fault trees to the more general problem of probability assessment of event trees. Our studies suggest that the traditional availability-based account does not fully explain pruning bias or the more general phenomenon of partition dependence. We propose an alternative mechanism: a judge begins with equal probabilities for all events to be evaluated and then adjusts this uniform distribution based on his or her beliefs about how the likelihoods of the events differ. Bias arises because the adjustment is typically insufficient. Although current best practices in subjective probability elicitation are designed to guard against availability and the other major causes of pruning bias that have been previously advanced in the literature, such best practices provide inadequate protection against a more pervasive tendency to anchor on equal probabilities. Understanding the nature and causes of partition dependence can help analysts identify conditions under which this bias may arise, predict conditions that may exacerbate or mitigate the effect, and develop more effective debiasing techniques.

In §2, we review literature on pruning bias and partition dependence. In §3, we describe a series of studies that document the robustness of partition dependence across a variety of contexts beyond fault trees, provide support for our interpretation of this phenomenon, and cast doubt on the necessity of alternative accounts that have been proposed to explain pruning bias. We close with a discussion of the interpretation and robustness of partition dependence, other manifestations of this phenomenon, and prescriptive implications of our results.

2. Literature Review

Fischhoff et al. (1978) presented professional automobile mechanics and laypeople with trees that identified several categories of reasons why a car might fail to start, as well as a residual category of reasons labeled "all other problems." Participants were asked to estimate the number of times out of 1,000 that a car would fail to start for each of the categories of causes specified. When the experimenters removed (pruned) specific categories of causes from the tree (e.g., battery charge insufficient) and relegated them to the residual category as in Figure 1, the judged probability of the residual category, as assessed by

Figure 1 Possible Reasons Why a Car Might Fail to Start



Note. Fischhoff et al. (1978) showed one group of participants the upper tree and another group the lower tree and asked participants to estimate the number of times out of 1,000 that a car would fail to start for a reason contained in each of the main categories.

a new a group of participants, did not increase by a corresponding amount. Instead, the probability from the pruned categories tended to be distributed across all of the remaining categories. Because the probability assigned to the residual category in the pruned tree was lower than the sum of probabilities of corresponding events in the unpruned tree, the pattern has subsequently come to be known as the *pruning bias* (e.g., Russo and Kolzow 1994).

Since the publication of Fischhoff et al. (1978), numerous authors have replicated and extended the basic result and proposed three major explanations for pruning bias: availability, ambiguity, and credibility. Below we review each of these accounts.

Availability. In explaining pruning bias, Fischhoff et al. (1978) invoked the availability heuristic (Tversky and Kahneman 1973): judged probabilities depend on the ease with which instances can be recalled or scenarios constructed. In the case of fault trees, explicitly mentioning a cause or category of causes will make that cause or category more salient, easing retrieval of related instances or construction of relevant scenarios, and hence leading to an increase in the corresponding judged probability. Support for such a mechanism has been provided by a number of researchers since Fischhoff et al. (1978), notably van der Pligt et al. (1987), Dubé-Rioux and Russo (1988), Russo and Kolzow (1994), and Ofir (2000).[1]

Ambiguity. Hirt and Castellan (1988) argued that some categories of problems in Fischhoff et al. (1978) are ambiguous. For example, suppose that the branch labeled "battery charge insufficient" were removed from the tree. Specific causes that might fit into that category, such as "faulty ground connection" or "loose connection to alternator," could just as well be assigned to a remaining branch labeled "ignition system defective" as to the residual "all other causes" category. Such ambiguous mapping of specific causes to categories could give rise to the observed pattern in which probabilities of pruned branches are distributed across remaining branches.

Credibility. A third explanation of the pruning bias is that people assume that a credible real-world fault tree would list enough possible causes so that the catchall category would be relatively unlikely, and each explicitly listed cause should have a nontrivial probability (Dubé-Rioux and Russo 1988, Fischhoff et al. 1978). This argument suggests that the pruning bias represents a demand effect (Clark 1985, Grice 1975, Orne 1962), whereby a participant considers the assessment as an implicit conversation with the experimenter in which the experimenter is expected to adhere to accepted conversational norms, including the expectation that any contribution should be relevant to the aims of the conversation. In the case of fault trees, the probability assessor may presume that any branch (other than the catchall) for which a probability is solicited must have a nontrivial probability; otherwise the probability of that item would be irrelevant, and therefore the query would violate conversational norms.

Although each of the three foregoing accounts (availability, ambiguity, and credibility) may contribute to some instances of pruning bias, previous studies suggest that the availability mechanism is most robust, contributing to pruning bias even in situations where the other mechanisms can be ruled out (Fischhoff et al. 1978, Russo and Kolzow 1994).[2] We assert, however, that even availability does not provide an adequate explanation of pruning bias. In particular, the availability account predicts that there should be little or no effect of pruning causes from a full tree if these causes are explicitly mentioned as part of the catchall category (so that the pruned causes are no longer out of sight even though their probabilities are not assessed separately). However, when Fischhoff et al. (1978) did this (Study 5), they nevertheless observed a strong pruning bias—a result that has received surprisingly little subsequent attention in the literature and that begs for a new interpretation of the phenomenon.

Anchoring and Insufficient Adjustment. We propose a fourth mechanism driving pruning bias: people anchor on a uniform distribution of probability

[1] Ofir (2000) noted that the original characterization of the availability heuristic (Tversky and Kahneman 1973) is that people sometimes judge likelihood by *ease* of retrieval (i.e., how readily instances come to mind) and not the *content* of retrieval (i.e., the number of instances retrieved; see Schwarz et al. 1991). The data of Ofir (2000) suggest that people with less domain knowledge rely on the ease with which they can retrieve specific causes (i.e., the availability heuristic), whereas people with more domain knowledge are influenced by the absolute number of specific causes that come to mind. Regardless of how an expert assesses likelihood (by ease of retrieval, content of retrieval, or some other mechanism), the availability-based account of pruning bias holds that specific causes or events are more likely to be considered when they are explicitly identified than when they are implicit constituents of a superordinate category.

[2] Fischhoff et al. (1978) cast doubt on the credibility account in their studies, because the mean probability assigned to the least important of seven branches was only 0.033, and the catchall category received a higher mean probability than the least probable identified category (Study 1). Russo and Kolzow (1994) experimentally manipulated the credibility of their trees by varying their alleged source, but found no evidence that this factor played a role in the observed pruning bias. They concluded that both ambiguity and availability contributed substantially to pruning bias for lay participants presented with a Fischhoff et al. (1978) automobile tree, but that availability was the only significant source of pruning bias for a second tree in which participants evaluated probabilities of various causes of death.

across all branches of the fault tree and adjust according to features that distinguish each branch. Because such adjustment is usually insufficient (Tversky and Kahneman 1974, Epley and Gilovich 2001), assessors are biased toward probabilities of $1/n$ for each of n branches in the tree. To illustrate, consider a fault tree consisting of seven branches plus a residual category. According to the anchoring account, the assessed probability of the residual will be biased toward 1/8 because it is one branch of eight. Now imagine pruning this tree so that three branches remain, plus a residual category. Although the residual subsumes five of the original eight branches, it now represents a single branch of four. The anchoring account predicts that the assessed probability of the residual in this pruned tree will be biased toward 1/4 rather than 5/8 and that the remaining branches will be biased toward 1/4 rather than 1/8.

Starting with equal probabilities for all branches can be interpreted as an intuitive application of the so-called principle of insufficient reason that has been attributed to Leibniz and Laplace (Hacking 1975). We say that a probability assessor adopts an *ignorance prior*, by which we mean a default judgment that branch probabilities are equal. Taking equal probabilities as a starting point, a probability assessor then adjusts (usually insufficiently) to account for his or her beliefs about how the likelihood of the events differ. Although we interpret this phenomenon in terms of anchoring and insufficient adjustment, a bias toward the ignorance prior may also be driven in some cases by enhanced accessibility of information that is consistent with an equal distribution of probability (Chapman and Johnson 2002) or the intrusion of error variance into the processing of frequency information (Fiedler and Armbruster 1994).

The anchoring hypothesis has not been extensively investigated, and the existing empirical evidence for it is rather indirect. Van Schie and van der Pligt (1990) asked undergraduates to estimate the proportion of acid rain that could be attributed to various causes and found that the cause "traffic" received a median rating of 14% in a (full) eight-branch tree and a median rating of 24% in a (pruned) four-branch tree, very close to the corresponding ignorance prior probabilities of 1/8 and 1/4, respectively. Johnson et al. (1991) asked undergraduates to judge the relative frequency of possible outcomes when a baseball player is at bat (e.g., single, double, out), the true values of which were known to the experimenters. Participants tended to underestimate relative frequencies when the corresponding ignorance prior was below the true value and overestimate relative frequencies when the corresponding ignorance prior was above the true value. Harries and Harvey (2000, pp. 441–442) obtained a similar result using a causes of death probability estimation task. Russo and Kolzow (1994, p. 26, footnote 13) asked participants "what should be" the probability of a residual category for a typical tree with different numbers (n) of labeled branches and observed that responses provided a "remarkable fit" to the formula $p_n = 1/(n+1)$, the ignorance prior.

In §3, we offer more direct evidence that pruning bias is driven by a tendency to allocate probability evenly across all events into which the state space happens to be partitioned. In five experiments we extend the observation of partition dependence from the narrow domain of fault trees (judgments of the relative frequency of various categories of fault in a system) to the more general domain of assessed probabilities of uncertain events. We demonstrate that even sophisticated probability assessors are susceptible to partition dependence in situations where the availability, ambiguity, and credibility mechanisms can be largely ruled out. Thus, we show that reliance on ignorance priors is the most robust source of partition dependence and that bias in subjective probability assessment may be more prevalent than has been previously supposed. Note that our results have important practical implications. To the extent that pruning bias is driven by the traditional mechanisms (availability, ambiguity, and credibility), existing best practices (e.g., conditioning experts, using the clarity test, and involving experts in the elicitation design) should mitigate the impact of these mechanisms and reduce the bias. However, to the extent that pruning bias is driven by a more general tendency to anchor on the ignorance prior, none of these best practices will be sufficient and new corrective procedures will be called for.

3. Experimental Evidence

Study 1: Separate Evaluation of Events Trumps Separate Description of Events

Most studies of fault trees have confounded whether or not particular causes were explicitly identified with whether participants were asked to assess probabilities of those causes. A straightforward reading of the availability account predicts that the probability assigned to a particular category will increase when it is explicitly identified in the tree but will not be affected by whether it is evaluated separately or with other causes. In contrast, the ignorance prior account predicts that the distribution of probabilities will be affected primarily by the number of branches that are explicitly evaluated. As mentioned earlier, some studies (including Experiment 5 of Fischhoff et al. 1978) have found that, holding descriptions constant, events are generally assigned higher probabilities when split into multiple branches that are evaluated separately. Likewise, in their account of judged probability, Rottenstreich and Tversky (1997) found that although unpacking a category (e.g., homicide) into a disjunction of subcategories (e.g., homicide by an acquaintance or homicide by a stranger) generally increases judged probability, separate assessment of the subcategories increases aggregate judged probabilities still further. A subsequent review of several studies (Sloman et al. 2004) found that the effect of separate evaluation is more robust and more pronounced than that of unpacking the description. This pattern is consistent with the notion that judged probabilities are affected more by a bias toward 1/2 for each event that is evaluated (1/2 is the ignorance prior when considering a target event against its complement) than by the enhanced availability of constituent events when the description is unpacked.

Our first study was designed to demonstrate in the context of event trees that the increase in probabilities because of separate evaluation (predicted by the ignorance prior account) persists even when the increase because of unpacking the description (predicted by the availability account) is negligible. Unlike previous fault tree studies cited above, we asked participants to judge the probabilities of future events, and we used well-defined categories whose constituents were well known to participants, rendering the ambiguity account less relevant.

Method. We recruited 93 weekend MBA students at Duke University midway through a required course on decision models. By the time the study was run, participants had already learned about basic decision analysis tools, including decision trees and subjective probability assessment methods. All participants had previously completed an MBA course on probability and statistics.

Participants judged probabilities that particular schools would receive the top spot in *Business Week's* next biennial ranking of business schools, a topic with which we expected them to be very familiar.[3] Each participant read the following instructions:

> In the most recent *Business Week* rankings of daytime MBA programs, the Wharton School was ranked #1. In each of the spaces provided below, please write your best estimate of the probability that the daytime MBA program(s) indicated will be ranked #1 in the next *Business Week* survey.... Please make sure that your probabilities sum to 100%.

Participants in the *full-tree* condition ($n = 30$) were then presented with a tree in which the strongest MBA programs (plus a catchall category) were listed alphabetically on separate branches:

- Chicago
- Harvard
- Kellogg
- Stanford
- Wharton
- None of the above

Participants in the *collapsed-tree* condition ($n = 32$) were presented with a tree in which the residual category had been unpacked to remind participants of the same schools:

- Chicago, Harvard, Kellogg, Stanford, or another school other than Wharton
- Wharton

Participants in the *pruned-tree* condition ($n = 31$) were presented a tree that included the following branches:

- A school other than Wharton
- Wharton

We predicted that unpacking the pruned tree into the collapsed tree would have a minimal effect on participants' judged probabilities of the residual category, because we would be reminding experts of schools that should be salient to them even without explicit prompting. However, we predicted that expanding the collapsed tree into the full tree would substantially increase the aggregate judged probability of schools other than Wharton because the ignorance prior increases from 1/2 to 5/6.

Results and Discussion. The results of Study 1 are displayed in Table 1 and accord with our predictions. The *pruned* and *collapsed* conditions both yielded median probabilities of 0.40 for the "other" (i.e., not Wharton) category. However, when asked to judge events separately in the *full* condition, the median sum of probabilities for schools other than Wharton jumps to 0.70. Based on a one-tailed, Wilcoxon

Table 1 **Results of Study 1**

Condition	Median P(School other than Wharton is ranked #1)	n	Significance levels using one-tailed, Wilcoxon rank-sum test: Comparing	p-value
Pruned	0.40	31	Pruned vs. collapsed	0.350
Collapsed	0.40	32	Pruned vs. full tree	0.005
Full tree	0.70	30	Collapsed vs. full tree	0.053

Note. The first column indicates the experimental condition. The second column lists the median judged probability (or sum of judged probabilities) that the next top-rated *Business Week* school would not be Wharton. The third column indicates the usable sample size. The fifth and sixth columns indicate significance levels of the designated statistical comparison between conditions.

[3] Fuqua administrators had previously conducted a survey of students admitted to Duke's daytime MBA program ($N = 285$), in which 99% of respondents indicated that they had used *Business Week* and/or *US News & World Report's* published rankings of business schools in deciding which business school to attend. Although our weekend MBA participants may have been somewhat less familiar with the details of the *Business Week* ranking than the daytime MBA students, we believe that our participants knew enough about this topic to make informed judgments in our study.

rank-sum statistic (which we use hereafter unless otherwise indicated), the median sum of judged probabilities for non-Wharton schools in the *full tree* is significantly different from median judged probabilities of the corresponding events in the *collapsed* and *pruned* conditions ($p = 0.05$ and $p = 0.005$, respectively). Judged probabilities for a school other than Wharton in the *collapsed* and *pruned* conditions do not differ significantly ($p = 0.35$).

The results for the school rankings replicate findings of Fischhoff et al. (1978) (Experiment 5) and Rottenstreich and Tversky (1997) that the judged probability of an event is higher when constituent events are assessed separately than when they are assessed as a single composite event. Furthermore, our results suggest that the availability-based account is not a necessary source of the pruning bias. In both the *pruned-tree* and explicit *collapsed-tree* conditions, for which schools other than Wharton comprise one of two branches, median judged probabilities were slightly below the ignorance prior of 1/2. In the separate evaluation (*full*) condition, for which schools other than Wharton comprise five of six branches, the median sum of probabilities is slightly below the ignorance prior of 5/6.

Study 2: Ignorance Gives Rise to Strong Partition Dependence

Decision and risk analysts strive to find knowledgeable experts to provide probability assessments. Of course, analysts must often obtain assessments concerning unfamiliar or unprecedented future events, for instance, in situations involving the development of a new technology or the management of an unproven hazard. The ignorance prior account suggests that partition dependence will be most pronounced in situations where probability assessors have little relevant knowledge, and therefore have little basis to adjust probabilities from the ignorance prior. In our second study, we asked business students to make judgments and decisions concerning the future closing value of the Jakarta Stock Index (JSX), a domain about which we expected them to know very little. We reasoned that if we could observe partition dependence for the JSX, it would be difficult to attribute this bias to an availability-based mechanism because the extension of our categories (i.e., the set of possible closing values to which each range refers) is readily apparent, and therefore unpacking into subranges will only remind participants of subcategories that were patently obvious in the original tree. Moreover, participants cannot easily judge likelihood by availability of instances because it is unlikely that these participants can recall any instance of closing values of the JSX. Of course, one could argue that judged probabilities under ignorance are arbitrary and not a valid measure of respondents' belief strength. To provide concomitant evidence that these judged probabilities accord with subjective degrees of belief, we also asked participants to make choices involving these events using an incentive-compatible payoff mechanism.

Method. Participants were 246 entering MBA students at Duke University who were asked during their orientation to complete a number of unrelated faculty research projects in exchange for a donation to a charity. All participants were presented with the following information:

> The JSX is the leading composite index of the Jakarta Stock Exchange. The closing value of the JSX on December 31 of this year will be in one of the following ranges:

Approximately half the participants were then presented with the following ranges:

(A) less than 500
(B) at least 500 but less than 1,000
(C) at least 1,000.

Participants in the *threefold low* condition ($n = 58$) were asked to judge the probability that the JSX would close in either range A or B. Participants in the *threefold high* condition ($n = 61$) were asked to judge the probability that the JSX would close in range C. The remaining participants were instead presented with the following ranges that entailed a refined partition of values above 1,000:

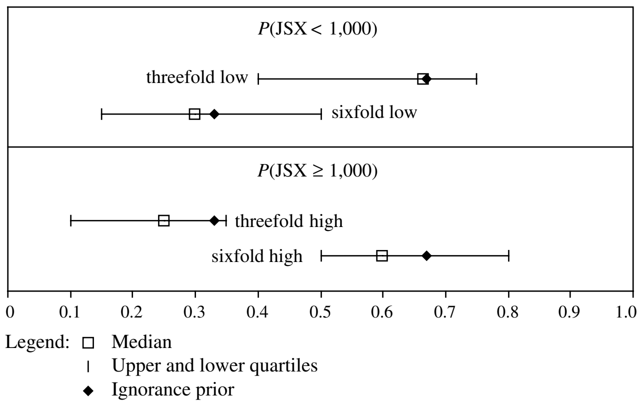
(a) less than 500
(b) at least 500 but less than 1,000
(c) at least 1,000 but less than 2,000
(d) at least 2,000 but less than 4,000
(e) at least 4,000 but less than 8,000
(f) more than 8,000.

Participants in the *sixfold low* condition ($n = 65$) were asked to judge the probability that the JSX would close in either range a or b. Participants in the *sixfold high* condition ($n = 62$) were asked to judge the probability that the JSX would close in range c, d, e, or f.

After providing a probability judgment, all participants were asked whether they would prefer to receive \$10 for sure or receive \$30 if the actual value of the JSX on the previous day had fallen into the specified interval (and receive nothing otherwise). We told participants that one respondent would be randomly selected to have his or her choice honored for real money.

Results and Discussion. Figure 2 displays the results of Study 2. Judged probabilities varied dramatically by experimental condition, consistent with the ignorance prior account. The median judged probability that the JSX would close below 1,000 was 0.67 in the *threefold low* condition (in which this event

Figure 2 Results of Study 2



Note. For each event and each partition condition, the figure depicts the ignorance prior probability, the median judged probability, and the interquartile range. "Threefold" and "sixfold" refer to the number of elements into which the state space was partitioned.

comprised two of the three specified ranges) but only 0.30 in the *sixfold low* condition (in which this event comprised two of six specified ranges), a significant difference ($p = 0.02$).

Similarly, the median judged probability that the JSX would close at 1,000 or above was 0.25 in the *threefold high* condition (in which this event comprised one of three specified ranges) and 0.60 in the *sixfold high* condition (in which this event comprised four of six specified ranges), again a significant difference ($p = 0.001$). In three of four conditions, judged probabilities did not differ significantly from the corresponding ignorance prior. Using binomial tests and distributing ties evenly, $p = 0.69$ in the *threefold low* condition, $p = 0.17$ in the *sixfold low* condition, $p = 0.04$ in the *threefold high* condition, and $p = 0.25$ in the *sixfold high* condition.

Results from the choice task echo the judged probabilities. A majority of participants in the *threefold low* condition (55%) indicated that they would rather receive \$30 if the JSX had closed below 1,000 than receive \$10 for sure, whereas a minority of participants in the *sixfold low* condition (31%) made the same choice ($\chi^2(1) = 7.48$, $p = 0.006$). Likewise, only 28% of *threefold high* participants indicated that they would rather receive \$30 if the JSX had closed at 1,000 or above, whereas 58% in the *sixfold high* participants made the same choice ($\chi^2(1) = 11.43$, $p = 0.001$).

Study 3: Domain Knowledge Moderates Partition Dependence

The first two studies establish that partition dependence can occur in situations where availability-based explanations are dubious at best. In the next study, we examine the extent to which domain knowledge moderates this phenomenon. Fischhoff et al. (1978), Ofir (2000), and Harries and Harvey (2000) show in the context of fault trees that the pruning bias is reduced but not eliminated as domain knowledge increases. The ignorance prior account implies more generally that increasing knowledge should be associated with less reliance on the ignorance prior distribution (i.e., more adjustment), and hence probabilities that are less partition dependent. We asked MBA students for probabilities relating to two domains for which we expected them to have very different levels of knowledge: starting salary of graduates from their program (a topic closely followed by MBA students) and the starting salary of Harvard Law graduates (a topic with which we expected them to be much less familiar).

Method. The participants in this study were 120 second-year MBA students at Duke University enrolled in an elective course in decision analysis. At the time of the study, these students had finished a first-year internship and were actively seeking permanent jobs. All participants had previously completed a course on probability and statistics and a course on decision models. The Duke MBA Career Services Office provides students with information about beginning salaries for graduates from previous classes.

The questionnaire asked participants to judge probabilities that the starting salary for a randomly chosen member of the current graduating class would fall into particular intervals. To construct roughly comparable partitions, we conducted a pretest in which a different sample of second-year MBA students assessed 10th, 50th, and 90th percentiles for the first-year starting salary of a randomly selected member of the graduating class of Duke MBA students and the same for the present graduating class of Harvard Law students. Based on these assessments, we created low and high partitions for both Duke MBA and Harvard Law salaries that were roughly comparable. Participants in the low (high) partition condition provided probabilities for both Duke MBA and Harvard Law salaries, in which low (high) salary ranges were broken into subranges, as displayed in Figure 3. In all cases, we counterbalanced the order in which the two sets of probabilities were elicited. As before, participants were asked to ensure that their assessed probabilities for each variable summed to 100%. In addition to the probability judgments, we asked participants to rate their level of knowledge of the two variables on a scale from 0 ("I know nothing") to 10 ("I know a great deal").

Results and Discussion. Median knowledge ratings were 7 for Duke MBA starting salaries (M = 6.78, SD = 1.87) and 2 for Harvard Law starting salaries (M = 2.03, SD = 2.00), confirming the validity of our a priori assumptions concerning relative knowledge.

Figure 3 Stimuli Used in Study 3

	Duke MBA	Harvard Law
Low partitions	\$55,000 or less ____ % \$55,001–\$65,000 ____ % \$65,001–\$75,000 ____ % \$75,001–\$85,000 ____ % - - - - - - - - - - More than \$85,000 ____ %	\$60,000 or less ____ % \$60,001–\$70,000 ____ % \$70,001–\$80,000 ____ % \$80,001–\$90,000 ____ % - - - - - - - - - - More than \$90,000 ____ %
High partitions	\$85,000 or less ____ % - - - - - - - - - - \$85,001–\$95,000 ____ % \$95,001–\$105,000 ____ % \$105,001–\$115,000 ____ % More than \$115,000 ____ %	\$90,000 or less ____ % - - - - - - - - - - \$90,001–\$105,000 ____ % \$105,001–\$115,000 ____ % \$115,001–\$130,000 ____ % More than \$130,000 ____ %

Note. Participants were asked to assess the probability that the starting salary for a randomly chosen member of the next graduating class would fall into each interval. (The dashed lines were not included in the questionnaire and are shown here solely to clarify the experimental design.) Each participant was presented with either the low partitions or high partitions for both schools.

In fact, only one person of 120 indicated more knowledge about Harvard Law than Duke MBA salaries; five others indicated the same degree of knowledge for both schools.

Table 2 presents results from Study 3. Before analyzing judged probabilities, we discarded responses from participants whose probabilities did not sum to 100%. The number of remaining responses for each cell is shown in Table 2. For a participant in the high partition, let $P_{\text{high}}(\text{Harvard} \le 90\text{K})$ denote the single judged probability that a randomly chosen graduate of Harvard Law will earn \$90,000 or less during his or her first year after graduation (the top entry in the lower right-hand cell in Figure 3). Let $P_{\text{low}}(\text{Harvard} \le 90\text{K})$ denote the corresponding sum of judged probabilities for a participant in the low partition (the sum of the top four entries in the upper right-hand cell in Figure 3). Define $P_{\text{high}}(\text{Duke} \le 85\text{K})$ and $P_{\text{low}}(\text{Duke} \le 85\text{K})$ similarly. Median probabilities presented in Table 2 reveal partition dependence for judgments of both Harvard and Duke salaries. In particular, judged probabilities are lower when they are derived from a single judgment than when they are derived from multiple judgments that are summed: $P_{\text{high}}(\text{Harvard} \le 90\text{K}) < P_{\text{low}}(\text{Harvard} \le 90\text{K})$ and $P_{\text{high}}(\text{Duke} \le 85\text{K}) < P_{\text{low}}(\text{Duke} \le 85\text{K})$. To perform an overall test for significance of the effect, we calculated $P_{\text{low}}(\text{Harvard} \le 90\text{K}) + P_{\text{low}}(\text{Duke} \le 85\text{K})$ for each participant in the low condition and $P_{\text{high}}(\text{Harvard} \le 90\text{K}) + P_{\text{high}}(\text{Duke} \le 85\text{K})$ for each participant in the high condition. The ignorance prior account predicts that the median sum of probabilities below the relevant cutoff ($\le$85K, $\le$90K) will be greater for participants in the low-partition condition than the corresponding probability for participants in the high-partition condition; this prediction is confirmed ($p < 0.0001$).

Table 2 Results of Study 3

	Duke MBA			Harvard Law		
Condition	$\le$\$85,000	$>$\$85,000	n	$\le$\$90,000	$>$\$90,000	n
Low partition	**0.75**	0.25	55	**0.75**	0.25	57
High partition	0.40	**0.60**	57	0.30	**0.70**	58

Note. The first column indicates experimental condition as illustrated in Figure 3. The second, third, fifth, and sixth columns list median judged probabilities of the designated events based on single judgments (lightface entries) and sums of the four separate judgments (bold entries). The fourth and seventh columns indicate the usable sample size for columns 2–3 and 5–6, respectively.

Thus, we observed substantial partition dependence for both schools, and this pattern was more pronounced for Harvard (difference of medians = 0.45) than for Duke (difference of medians = 0.35). To test the statistical significance of the interaction, we calculated $P_{\text{low}}(\text{Harvard} \le 90\text{K}) - P_{\text{low}}(\text{Duke} \le 85\text{K})$ among participants in the low-partition conditions (for whom these values were the sums of four separate judgments) and $P_{\text{high}}(\text{Harvard} \le 90\text{K}) - P_{\text{high}}(\text{Duke} \le 85\text{K})$ among the participants in the high-partition condition (for whom these values each refer to a single judgment). If partition dependence is more pronounced for the Harvard Law judgments than for the Duke MBA judgments, we would expect the difference to be larger for the low-partition respondents than for the high-partition respondents. This difference of differences approaches significance by a one-tailed Wilcoxon test ($p = 0.12$) and by a t-test ($t(105) = 1.63$, $p = 0.05$).

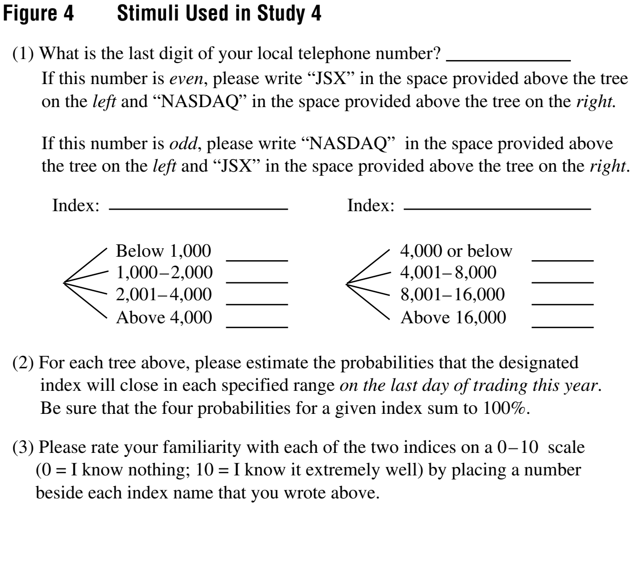
Study 4: Credibility and Demand Effects Do Not Adequately Explain Partition Dependence

In Studies 1–3, we observed partition dependence in situations where availability and ambiguity mechanisms were unlikely to have played a role. We also found that the effect was especially pronounced for less familiar domains, precisely the situation where greater reliance on the ignorance prior might be expected. As mentioned earlier, one might be tempted to dismiss partition dependence, like the pruning bias, as a demand effect (Clark 1985, Grice 1975, Orne 1962) or "credibility" effect (Dubé-Rioux and Russo 1988). According to this interpretation, probability assessors derive information from the tree they have been presented by assuming that it is a credible event tree in which each (nonresidual) branch has a nontrivial probability of occurrence. Hence, one might worry that the pruning bias and our demonstrations of partition dependence could be explained as an experimental artifact that would not appear if participants knew that the particular partitions they saw were arbitrary. In Studies 1–3, we have attempted to minimize these effects by using familiar domains (MBA program rankings in Study 1, MBA salaries in Study 2) and financial incentives (Study 2). To address this issue more directly, we designed a new study in which participants could clearly see that they had been randomly assigned to a particular partition condition. An observation of partition dependence in this study would provide stronger evidence that the credibility/demand-effect explanation cannot fully account for the phenomenon.

Method. We recruited 102 students enrolled in a decision models course in Duke's Weekend Executive MBA program. Four participants were selected at random to receive $20 as a reward for completing a brief survey. Figure 4 displays the stimuli presented in the questionnaire. All participants were presented with two different four-interval partitions: the low partition had three intervals up to 4,000 and one interval above 4,000, and the high partition had one interval up to 4,000 and three intervals above 4,000.

Each participant assigned himself or herself to an experimental condition based on the last digit of his or her local telephone number. If the number was even (odd), the participant was asked to write "NASDAQ" above the left (right) tree and "JSX" above the right (left) tree. The order of presentation of low and high partitions was counterbalanced. Ultimately, $n = 53$ participants assigned themselves to the NASDAQ low, JSX high-partition condition, and $n = 49$ participants assigned themselves to the NASDAQ high, JSX low-partition condition. Below these trees, we instructed participants to assign probabilities that the designated index would close in the specified range on the last day of trading of the present year. Participants were asked to ensure that their probabilities for each index summed to 100%. Finally, participants were asked to indicate their familiarity with each index on a 0–10 scale.

Figure 4 **Stimuli Used in Study 4**



It should have been apparent to participants that they had assigned themselves at random to partition conditions. To the extent that participants read anything into the particular pair of partitions they saw, they might have noticed the common 4,000 threshold (at the time of the study, the NASDAQ index was closing near 4,000). Under the credibility account, noticing the common threshold might have biased probabilities above and below 4,000 toward 50%. However, this would also drive responses toward consistency across conditions and away from partition dependence. Thus, our design provides a conservative test of partition dependence in a situation where credibility and demand effects are not likely to play a role.

Results and Discussion. Table 3 summarizes the results of Study 4. Median knowledge ratings were 7 for NASDAQ (M = 6.34, SD = 2.67) and 0 for the JSX (M = 0.18, SD = 0.77), confirming our a priori expectation that the NASDAQ would be much more familiar than the JSX. Of 92 participants who provided knowledge ratings, only one rated the same level of knowledge for both NASDAQ and JSX (both zero). All others reported higher knowledge for NASDAQ.

Before analyzing the data, we discarded responses that did not sum to 100% for each event tree. The number of responses remaining in each cell is shown in Table 3. First, consider the results for the full data set shown in the top section of Table 3. The overall pattern of partition dependence is highly significant ($p < 0.0001$), and it is significant for both the NASDAQ ($p = 0.02$) and JSX ($p = 0.003$) taken separately. The results for JSX closely replicate the results from Study 2. In fact, the median probability for all four intervals for JSX was 25% in both low- and high-partition conditions. More strikingly, modal

Table 3 **Results of Study 4**

		NASDAQ			JSX		
Sample	Condition	≤4,000	>4,000	*n*	≤4,000	>4,000	*n*
Full data set	Low partition	**0.50**	0.50	49	**0.75**	0.25	52
	High partition	0.25	**0.75**	51	0.25	**0.75**	45
NASDAQ "experts"	Low partition	**0.25**	0.75	24	**0.75**	0.25	24
	High partition	0.32	**0.70**	24	0.25	**0.75**	23
NASDAQ "nonexperts"	Low partition	**0.80**	0.20	21	**0.75**	0.25	22
	High partition	0.12	**0.88**	22	0.23	**0.77**	18

Note. The first column indicates the relevant sample ("experts" and "nonexpert" subsamples were segregated based on a median split of knowledge ratings). The second column indicates experimental condition as illustrated in Figure 4 ("low partition" refers to the condition in which the event ≤4,000 consisted of three intervals and "high partition" refers to the condition in which the event >4,000 consisted of three intervals). The third, fourth, sixth, and seventh columns list median judged probabilities of the designated events based on single judgments (light-face entries) and sums of the three separate judgments (bold entries). The fifth and eighth columns indicate the usable sample size for columns 3–4 and 6–7, respectively.

judged probabilities under ignorance coincided precisely with the ignorance prior: 42% of respondents (41 of 97) provided equal probabilities for the four JSX intervals, and these responses were distributed roughly evenly across partition conditions. In light of the obvious random assignment of participants to experimental condition, these results indicate that partition dependence cannot easily be dismissed as a demand effect.

As in Study 3, rated knowledge seemed to moderate partition dependence. The difference between probabilities in high- and low-partition conditions was less pronounced for NASDAQ (difference in medians $= 0.25$) than for JSX (difference in medians $=$ 0.50), and this interaction is highly significant ($p =$ 0.002). To explore this knowledge effect further, we split the sample based on participants' NASDAQ knowledge ratings. Recall that the median rating was 7. Table 3 shows median judged probabilities among the "experts" with knowledge ratings of 7 or higher and "nonexperts" with knowledge ratings below 7. Partition dependence is extremely pronounced among the nonexperts (difference in medians $= 0.68$, $p = 0.002$) but disappears among the experts, (difference in medians $= -0.07$, $p = 0.44$), and the interaction is highly significant ($p = 0.001$).[4] There is no difference, however, between NASDAQ experts and nonexperts on JSX assessments. These results provide further support for the claim that increasing knowledge leads to a decreasing reliance on the ignorance prior. Although the self-rated experts concerning the NASDAQ did not fall prey to partition dependence in this instance, we suspect that this result is the exception rather than the rule, given the observations of partition dependence in other studies of participants with considerable substantive knowledge (e.g., Study 1, Study 3, the auto mechanics of Fischhoff et al. 1978, Ofir 2000). We also suspect that the juxtaposition of probability trees in Study 3 may have artificially cued some more savvy participants to take pains to be consistent in their use of probabilities across the two trees with which they happened to be presented.

Study 5: Professional Decision Analysts Are Not Immune

The results of Studies 1–4 demonstrate the robustness of partition dependence among fairly sophisticated participants: graduate students of business, most of whom had training in probability, statistics, and decision models (Studies 1, 3, and 4), and some of whom had additional training in decision analysis (Study 3). Despite this procedural sophistication, one could argue that the participants in our studies did not have extensive training in and experience using probability assessment methods, and that such training and experience might eliminate the bias. In our final study, we address this issue by replicating the method of Study 4, using a population with extensive training in decision analysis: members of the Decision Analysis Society (DAS) of INFORMS.

Method. We solicited responses from 169 members of the DAS e-mail list, obtaining 58 responses. We asked respondents to judge probabilities concerning the membership totals of DAS and the Society of Quantitative Analysts (SQA) five years in the future. We told participants that the SQA is "...a professional society incorporated in 1989 that is concerned with the application of innovative quantitative techniques in finance, investment, and risk management." At the time of the survey, the DAS tallied 764 registered members, and the SQA reported on its website "over 200 members," though we did not mention either of these facts in the instructions.

The first part of the survey followed exactly the same design as Study 4 in which we asked participants to assign themselves to conditions (low and high partitions), using the last digit of their primary home telephone number. The low partition included the following ranges: 400 or less, between 401 and 600, between 601 and 800, between 801 and 1,000, and more than 1,000. The high partition included the following ranges: 1,000 or less, between 1,001 and 1,200, between 1,201 and 1,400, between 1,401 and 1,600, and more than 1,600. The order of presentation of low and high partitions was counterbalanced. We obtained $n = 30$ participants in the DAS low, SQA high-partition condition and $n = 28$ participants in the DAS high, SQA low-partition condition. Participants then assessed the probabilities that the total membership of the DAS and SQA would fall into designated ranges. As usual, we asked participants to verify that their probabilities summed to one for each tree, and we counterbalanced the order of the trees. Following the probability assessments, we asked each participant for his or her highest level of education completed (e.g., BA, MS, ABD, PhD); whether he or she had taught a course in decision analysis; the number of applied decision or risk analysis projects in which he or she had elicited probabilities over the previous two years; and (if more than zero) what elicitation techniques were used.

Results and Discussion. Our respondents ($N = 57$ usable responses) collectively represent considerable

[4] To test the significance of the interaction, we considered the event NASDAQ $\leq 4{,}000$. For this event, we pooled responses from low-partition experts and high-partition nonexperts, and likewise we pooled responses from high-partition experts and low-partition nonexperts. An interaction implies unequal medians for these two pooled groups. We used the Kruskal-Wallis statistic to test for differences in the medians of the two groups.

Table 4 **Results of Study 5**

	DAS			SQA		
Condition	≤1,000	>1,000	*n*	≤1,000	>1,000	*n*
Low partition	**0.90**	0.10	29	**0.80**	0.20	27
High partition	0.65	**0.35**	28	0.52	**0.48**	28

Note. The first column indicates experimental condition ("low partition" refers to the condition in which the event ≤1,000 consisted of four intervals and "high partition" refers to the condition in which the event >1,000 consisted of four intervals). The second, third, fifth, and sixth columns list median judged probabilities of the designated events based on single judgments (light-face entries) and sums of the four separate judgments (bold entries). The fifth and eighth columns indicate the usable sample size for columns 2–3 and 5–6, respectively.

decision analysis expertise: 86% had PhDs, 75% had taught at least one course in decision analysis, and 63% had elicited probabilities in a total of 156 applied decision analysis projects in the previous two years.

Table 4 summarizes the results of Study 5. Note that some participants chose not to provide all of the probabilities requested; the number of usable responses for each cell is shown in Table 4. The responses showed significant partition dependence overall ($p < 0.0001$), and this pattern is significant for judgments of DAS (difference in medians $= 0.25$, $p = 0.01$). The effect size was similar for SQA (difference in medians $= 0.25$). Because responses for the latter society exhibited a great deal of noise, the pattern did not achieve statistical significance by a nonparametric, Wilcoxon rank-sum test ($p = 0.45$), though it was significant by a parametric t-test ($t(44) = -2.28$, $p = 0.01$).

To explore the robustness of our results, we examined a subsample of the 25 decision analysts with PhD degrees who had worked on at least one applied decision analysis project in the previous two years and had also taught at least one decision analysis course. Table 5 displays the results of this analysis. Not surprisingly, the overall effect of partition dependence is somewhat smaller among these superexperts, but the effect nevertheless approaches statistical significance (the difference in medians was 0.10 for DAS and 0.22 for SQA, overall $p = 0.05$ by one-tailed, Wilcoxon rank-sum test).

Our main purpose in Study 5 was to demonstrate the robustness of partition dependence among a sample of participants with high procedural expertise.

Table 5 **Internal Analysis of Study 5**

	DAS			SQA		
Condition	≤1,000	>1,000	*n*	≤1,000	>1,000	*n*
Low partition	**0.88**	0.12	12	**0.80**	0.20	12
High partition	0.73	**0.27**	12	0.58	**0.42**	12

Note. This table presents the same analysis as Table 4 for the most experienced decision analysts in the sample.

Although one might expect DAS members to know more about the size of DAS than SQA, our results here reveal no significant knowledge effect ($p = 0.30$), though there was a nonsignificant tendency among the superexperts. We did not collect knowledge ratings regarding the two societies, but we speculate that the lack of a significant knowledge effect may stem from small differences in knowledge across domains: We provided some information concerning the SQA, gave no information about current membership for either society, and asked about membership of both societies five years in the future.

4. General Discussion

In this paper, we have extended the analysis of pruning bias from fault trees to the more general phenomenon of partition dependence in assessing subjective probability. In five studies, we have accumulated support for the notion that this phenomenon is driven primarily by a bias toward equal allocation of probability across all events into which the state space is partitioned, rather than the enhanced availability of events that happen to be made explicit, ambiguity of event categories, or information unintentionally conveyed by the particular branches that are selected for evaluation. In all of our studies, we minimized ambiguity effects by using simple, well-defined event trees. In Study 1, we showed that unpacking the description of an event (a school other than Wharton will be the next top-rated business school) into its most obvious constituents (Chicago, Harvard, Kellogg, Stanford, or another school) did not lead to an increase in judged probability; however, asking participants to assess constituents separately gave rise to a dramatic increase in aggregate probability. This result suggests that partition dependence was driven by the number of branches evaluated rather than the availability of categories being evaluated. In subsequent studies, we asked participants to provide probabilities for state spaces defined by continuous random variables. In these cases, participants were not required to recall instances of categories, hence availability was unlikely to play a role. In Study 2, participants displayed a pronounced degree of partition dependence in a situation where they were unlikely to have much knowledge (future close of the JSX), with judged probabilities very close to corresponding ignorance prior probabilities, and this tendency was also reflected in betting behavior. Study 3 demonstrated that partition dependence among Duke MBA students was more pronounced for a relatively unfamiliar domain (Harvard Law graduate salaries) than a relatively familiar domain (Duke MBA graduate salaries). In our final two studies, we explicitly addressed the concern that our choice of partitions might have conveyed information to participants, because we provided all participants with multiple partitions and made it clear to them that they had been assigned at random to experimental conditions. Study 4 replicated and strengthened the major findings of Studies 2 and 3 for judgments concerning the future close of familiar and unfamiliar stock indices, and Study 5 demonstrated partition dependence among participants with considerable procedural expertise: members of the DAS. We close with a discussion of the interpretation and robustness of partition dependence, manifestations of partition dependence outside the domain of event trees, and prescriptive implications for decision and risk analysis.

The Interpretation and Robustness of Partition Dependence

Partition dependence refers to the tendency for judged probabilities to vary systematically with the way a state space is partitioned into events for which probabilities are assessed. One could argue that this phenomenon is analogous to framing effects in studies of risky choice (Kahneman and Tversky 1984, Tversky and Kahneman 1986) in which decisions are influenced by the way in which alternatives are described (e.g., in terms of losses and gains relative to a reference point). As with framing effects, respondents seem to accept the partition that is suggested to them in the form of an event tree, and they seem to be somewhat insensitive to the arbitrary nature of this partition. Our interpretation of this phenomenon is that people anchor their judgments on equal probabilities for each event in the specified partition (the ignorance prior distribution) and adjust insufficiently to account for their beliefs about how the likelihood of the events differ. This said, we acknowledge that in some elicitation contexts, other mechanisms may also contribute.

Consistent with our anchoring-and-adjustment account, we found that participants with greater substantive expertise show less partition dependence, and the effect may sometimes disappear when participants are particularly knowledgeable, especially if they consider multiple partitions of the state space (Study 4). However, we suspect that in many contexts, experts may lack sufficient knowledge to overcome the bias. For instance, we believe that the MBA students in Study 3 were more knowledgeable about their future salaries than any other population would have been without explicit statistics at hand, and the knowledge ratings of these de facto experts confirmed a subjective feeling of high expertise. Perhaps more striking, partition dependence seems to be quite robust to varying levels of procedural expertise. It is difficult to imagine a population with greater knowledge of subjective probability assessment techniques than the DAS members surveyed in Study 5, yet even the most expert among them fell prey to partition dependence.

Other Manifestations of Partition Dependence

Partition dependence has been observed not only in the context of event trees (in which assessors judge the probabilities of a number of exclusive and exhaustive events) but also in simple probability judgment. Fox and Rottenstreich (2003) demonstrated that the language of a probability query can facilitate either a twofold "case" partition {the target event obtains, the target event fails to obtain} and a corresponding ignorance prior of 1/2 or an n-fold "class" partition {event 1 obtains, event 2 obtains, ... event n obtains} and a corresponding ignorance prior of $1/n$. For example, participants who were asked to judge the probability that "The temperature on Sunday will be higher than every other day next week" gave responses that tended toward 1/2, whereas participants who were asked to judge the probability that "Next week, the highest temperature of the week will occur on Sunday" gave responses that tended toward 1/7. See Fox et al. (2005) demonstrated partition dependence in a learning environment where participants observed colored shapes that flashed on a computer screen with varying relative frequencies. When participants were then asked to judge the probability of a particular attribute (e.g., the probability of a black object versus the probability of a triangle), they were biased toward the ignorance prior probability defined by the number of possible values that the target attribute could take (black was one of two possible colors, while a triangle was one of four possible shapes) even when these attributes appeared with identical objective frequencies. This bias diminished but did not disappear when participants had more extensive opportunities to learn the distribution of objects. Finally, Fox and Levav (2004) showed that common mistakes solving conditional probability puzzles, such as the "Monty Hall" problem, may reflect naïve extensional reasoning in which people subjectively partition the state space on the basis of initial conditions, edit the partition using conditioning information, and calculate probability as a ratio of remaining events in the partition. They show further that subtle rewording of these problems can facilitate the use of more appropriate partitions and a higher frequency of correct responses.

Partition dependence has been observed not only in likelihood judgment but also in other domains relevant to decision analysis. Weber et al. (1988) reported that when people are asked to assign weights to different attributes of potential outcomes, they assign greater overall weight if an attribute is split into component parts and weights are assessed separately for each component. This is consistent with a tendency to spread out weight relatively evenly among the attributes that happen to be identified. Benartzi and Thaler (2001) showed that when people make allocations of retirement savings among potential investments in defined contribution plans, they tend to diversify naively among the available options. For example, people offered a stock fund and a bond fund typically allocate half of their savings to each fund, while people offered a stock fund and a mixed stock/bond fund also typically allocate half of their savings to each fund. Langer and Fox (2005) extended these results to allocation among simple chance lotteries and also find that allocations varied with the hierarchical organization of options (e.g., the grouping of investments by vendor), which apparently influenced how the set was subjectively partitioned. Fox et al. (forthcoming) extended the observation of partition dependence to riskless allocation of resources to beneficiaries (financial aid recipients or charities) and consumption opportunities to time periods. For a review of various manifestations of partition dependence in decision analysis, managerial resource allocation, and consumer choice, see Fox et al. (2005).

The foregoing examples of partition dependence all involve the allocation of some scarce resource (probability, attribute weight, or money) over a fixed set of possibilities (events, attributes, or investments). All reflect a bias toward even allocation of the resource across the specified possibilities. These should be distinguished from superficially similar cases in which the weight assigned to an event depends on its rank relative to other events in the partition. For example, Birnbaum (2004) explains some event-splitting effects in risky choice (Starmer and Sugden 1993) with his "transfer of attention exchange" (TAX) model, in which decision weight is transferred from higher-valued to lower-valued outcomes. Similarly, Windschitl and Wells (1998) report that subjective likelihood of a focal event increases when the most likely alternative outcome is split into several less likely constituents.

Prescriptive Implications

In our survey of DAS members for Study 5, we asked what techniques these experts used in applied probability elicitation projects involving continuous variables. Respondents reported that 58% of the time they rely on assessments based on prespecified intervals (where intervals are either provided by the analyst or suggested by the expert) such as those used in this study. Another predominant technique is to elicit fractiles for the uncertain variable (often 10th, 50th, and 90th percentiles), a method that typically yields an overconfidence bias (e.g., confidence intervals for which the true value of the variable in question lies below the 10th percentile or above the 90th percentile more than 20% of the time; see Lichtenstein et al. 1982, Klayman et al. 1999). Further research is needed to determine whether some manifestation of partition dependence is observed in fractile elicitation. But it is clear that both of these elicitation methods are susceptible to strong and persistent biases.

The present work has significant implications for improving existing best practices for eliciting subjective probabilities. Building on Russo and Kolzow's (1994) process account, we suggest that assigning probabilities to event trees entails three subtasks, each of which may be susceptible to distinct forms of bias. First, experts must *interpret* the extension of each event to be evaluated—to what kinds of events does each branch refer? This stage may entail both the generation of possible constituent events and categorization of constituent events to branches of the tree. For instance, if asked to assess the probabilities that a randomly selected death is because of "disease," "accident," "homicide," or "suicide," an expert might need to subjectively elaborate the category "disease" by noting that it includes heart attacks, cancer, strokes, and various other diseases. Second, experts must *evaluate support* for each elementary event using judgmental heuristics, explicit arguments, computational models, historical frequencies, or some other approach. For instance, a driver may assess the relative likelihood of various kinds of car failures by how easily instances of each category come to mind. Third, experts must *map this impression of relative support onto a set of numbers* that sum to one. This decomposition organizes mechanisms that may contribute to partition dependence and points to specific corrective procedures that target each subtask.

Biases located at the first stage (interpretation of categories) may be driven by the availability and ambiguity mechanisms. Thus, one would expect these biases to play a greater role for state spaces partitioned by category (e.g., different causes of death) than for state spaces partitioned into continuous intervals (e.g., closing stock values) in which the interpretation of categories is transparent. For categorical trees, the analyst can minimize this form of bias by working closely with the expert to carefully define and elaborate the interpretation of each branch. Thus, conditioning of experts (e.g., Merkhofer 1987) draws out extensive knowledge about the topic at hand and should reduce availability effects. Use of the clarity test (Howard 1988) is designed explicitly to banish ambiguity of categories from the assessment task.

Biases located at the second stage (assessment of support for each branch) can arise from a variety of sources that vary with the reasoning invoked by the expert. For instance, the assessor may invoke judgmental heuristics that give rise to bias (Kahneman et al. 1982, Gilovich 2002) or may make inappropriate assumptions about the distribution of support in a credible tree. The analyst may be able to guard against such biases to some extent by inducing the expert to articulate his or her reasoning, assumptions, and sources of information. Furthermore, if the analyst works with the expert to develop an appropriate partition in the first place, the expert should be more likely to provide judgments that reflect the expert's genuine knowledge of the event and less liable to "second guess" the analyst's beliefs about the relative likelihood of each branch.

Biases located at the third stage (mapping assessed support onto a set of numbers) include a tendency to anchor on the ignorance prior; such biases may be the most resistant to correction because they are least amenable to conscious reflection (cf. Arkes 1991, Larrick 2004). To minimize this bias, we suggest that the analyst should strive to direct the expert's attention across the state space in as evenhanded a manner as possible. For categorical partitions, it will often be possible to tell whether one partition is more evenhanded than another. For instance, in judging the probability that one's firm will win a competitive bid against a large number of competing firms, it may be convenient to assess the probabilities that (1) one's own firm will win, and (2) any one of the competing firms will win. However, we suspect that most people would consider a partition in which each competing firm's chances are evaluated separately to be more evenhanded.

For continuous variables, it may be more difficult to determine what is an evenhanded partition. For instance, consider the decision of whether to launch a satellite on a particular day. Success may depend on the ambient temperature T, and it may be convenient to ask experts to assess probabilities that T will fall above or below a specific target value (e.g., $T \leq 0°$ C versus $T > 0°$ C. However, asking experts to assess probabilities that T will fall in various specified intervals (e.g., $T \leq 0°$ C, $0°$ C $< T \leq 5°$ C, $5°$ C $< T \leq 10°$ C, $T > 10°$ C) may lead them to consider a more complete range of possible temperatures. Unfortunately, there may be little consensus concerning which set of intervals are the most evenhanded for most continuous variables.

Probability elicitation procedures used in decision and risk analysis (Clemen and Reilly 2001, Keeney and von Winterfeldt 1991, Morgan and Henrion 1990, Spetzler and Staël Von Holstein 1975, von Winterfeldt and Edwards 1986) can be thought of as instructions and devices to encourage deliberate and conscious reasoning. We paraphrase such best practice (described in detail in the references above) as, "Elicit probabilities in a variety of ways and ask the expert to reconcile the inevitable inconsistencies among his or her judgments." In particular, Spetzler and Staël Von Holstein (1975) describe several different approaches for assessing probability distributions for continuous variables, including fixing a value and asking for a cumulative (or exceedance) probability at that value; specifying a probability and asking for the corresponding fractile; asking for range estimates (e.g., 10th and 90th percentiles); and using the interval-splitting method. They show how the results from such a set of questions can lead to inconsistent probabilities, indicating the need to have the expert reconcile these differences. Responses from our survey of decision analysis experts in Study 5 are encouraging in this respect: nearly half (49%) of these experts indicated that they sometimes use multiple methods for eliciting subjective probabilities, and 12% indicated that they always use at least two methods.

Our results suggest an extension of Spetzler and Staël Von Holstein's (1975) advice to include the use of multiple partitions as well as multiple assessment methods. Using multiple partitions can highlight inconsistencies that may arise because of reliance on different ignorance priors, and the analyst can then help the expert recognize and reconcile those inconsistencies. The particular partitions that should be used will naturally vary case by case. Thus, it may make sense for the analyst and expert to work together to specify several different partitions and explicitly compare results.

We note, however, that consistency is only one criterion by which to evaluate the quality of an expert's subjective probabilities. Others include calibration and resolution (see, for example, Yates 1990). Scoring rules are often used to evaluate probabilities, and the Brier score (Brier 1950) in particular, can be broken down into calibration and resolution components. Further research is needed to determine whether using multiple partitions can lead to improvements in these scores. Although we would expect to see some improvement simply because of the reconciliation of inconsistencies, the extent to which the use of multiple partitions improves calibration and resolution is an open question.

An alternative approach is to debias the judgments rather than the judge; doing so requires an explicit model of anchoring on the ignorance prior. Fox and Rottenstreich (2003), for example, introduce a model that is a refinement of support theory (Tversky and Koehler 1994) in which judged probabilities are a compromise between the balance of support for the target hypothesis (relative to its complement) and the ignorance prior. An alternative model is given by Clemen and Ulu (2005). One could in principle estimate the parameters of such a model and back out subjective probabilities that are untainted by reliance on the ignorance prior. Further empirical work is necessary to determine the feasibility of implementing this approach in decision analysis practice.

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